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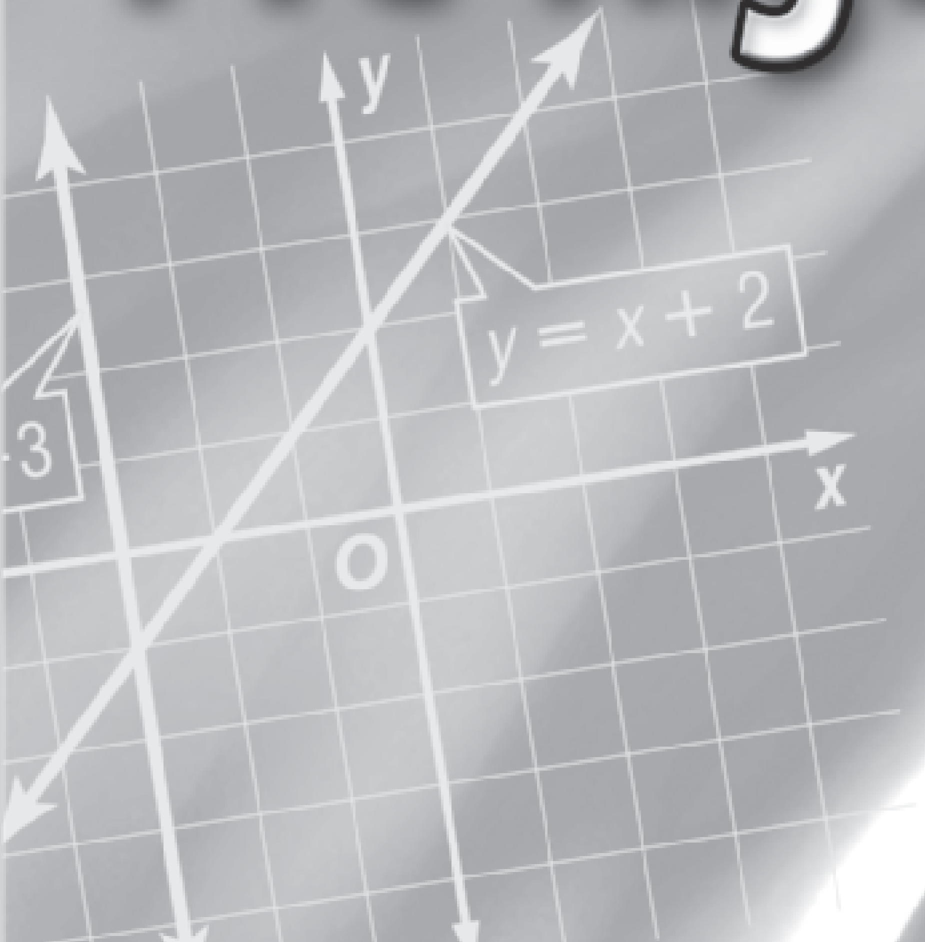
Study Guide and Intervention Workbook

Pre-Algebra



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To the Student

This *Study Guide and Intervention Workbook* gives you additional examples and problems for the concept exercises in each lesson. The exercises are designed to aid your study of mathematics by reinforcing important mathematical skills needed to succeed in the everyday world. The materials are organized by chapter and lesson, with two Study Guide and Intervention worksheets for every lesson in *Glencoe Pre-Algebra*.

Always keep your workbook handy. Along with your textbook, daily homework, and class notes, the completed *Study Guide and Intervention Workbook* can help you in reviewing for quizzes and tests.

To the Teacher

These worksheets are the same ones found in the Chapter Resource Masters for *Glencoe Pre-Algebra*. The answers to these worksheets are available at the end of each Chapter Resource Masters booklet as well as in your Teacher Edition interleaf pages.

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Study Guide and Intervention Workbook, Pre-Algebra

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Contents

1-1 Study Guide and Intervention

Words and Expressions

Translate Verbal Phrases into Expressions A **numerical expression** contains a combination of numbers and operations such as addition, subtraction, multiplication, and division. Verbal phrases can be translated into numerical expressions by replacing words with operations and numbers.

+	−	×	÷
plus	minus	times	divide
the sum of	the difference of	the product of	the quotient of
increased by	decreased by	of	divided by
more than	less than		among

Example **Write a numerical expression for each verbal phrase.**

a. the product of seventeen and three

Phrase the **product** of seventeen and three

Expression 17×3

b. the total number of pencils given to each student if 18 pencils are shared among 6 students

Phrase 18 shared **among** 6

Expression $18 \div 6$

Exercises

Write a numerical expression for each verbal phrase.

1. eleven less than twenty

2. twenty-five increased by six

3. sixty-four divided by eight

4. the product of seven and twelve

5. the quotient of forty and eight

6. sixteen more than fifty–four

7. six groups of twelve

8. eighty–one decreased by nine

9. the sum of thirteen and eighteen

10. three times seventeen

1-1 Study Guide and Intervention *(continued)*

Words and Expressions

Order of Operations Evaluate, or find the numerical value of, expressions with more than one operation by following the **order of operations**.

Step 1 Evaluate the expressions inside grouping symbols.

Step 2 Multiply and/or divide from left to right.

Step 3 Add and/or subtract from left to right.

Example **Evaluate each expression.**

a. $6 \cdot 5 - 10 \div 2$

$6 \cdot 5 - 10 \div 2 = 30 - 10 \div 2$ Multiply 6 and 5.

$= 30 - 5$ Divide 10 by 2.

$= 25$ Subtract 5 from 30.

b. $4(3 + 6) + 2 \cdot 11$

$4(3 + 6) + 2 \cdot 11 = 4(9) + 2 \cdot 11$ Evaluate (3 + 6).

$= 36 + 22$ Multiply 4 and 9, and 2 and 11.

$= 58$ Add 36 and 22.

c. $3[(7 + 5) \div 4 - 1]$

$3[(7 + 5) \div 4 - 1] = 3[12 \div 4 - 1]$ Evaluate (7 + 5) first.

$= 3(3 - 1)$ Divide 12 by 4.

$= 3(2)$ Subtract 1 from 3.

$= 6$ Multiply 3 and 2.

Exercises

Evaluate each expression.

1. $6 + 3 \cdot 9$

2. $7 + 7 \cdot 3$

3. $14 - 6 + 8$

4. $26 - 4 + 9$

5. $10 \div 5 \cdot 3$

6. $22 \div 11 \cdot 6$

7. $2(6 + 2) - 4 \cdot 3$

8. $5(6 + 1) - 3 \cdot 3$

9. $2[(13 - 4) + 2(2)]$

10. $4[(10 - 6) + 6(2)]$

11. $\dfrac{(67 + 13)}{(34 - 29)}$

12. $6(4 - 2) + 8$

13. $3[(2 + 7) \div 9] - 3$

14. $(8 \cdot 7) \div 14 - 1$

15. $\dfrac{4(18)}{2(9)}$

16. $(9 \cdot 8) - (100 \div 5)$

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1-2 Study Guide and Intervention

Variables and Expressions

Translate Verbal Phrases An **algebraic expression** is a combination of variables, numbers, and at least one operation. A **variable** is a letter or symbol used to represent an unknown value. To translate verbal phrases with an unknown quantity into algebraic expressions, first define the variable.

Algebraic Expressions

The letter x is most often used as a variable. → $x + 3$

$7d$ means $7 \times d$. mn means $m \times n$. → $7d - 2$, mn

$\frac{b}{5}$ means $b \div 5$. → $\frac{b}{5}$

Example **Translate each phrase into an algebraic expression.**

a. five inches longer than the length of a book

Words five inches longer than the length of a book
Variable Let b represent the length of the book.
Expression $b + 5$

b. two less than the product of a number and eight

Words two less than the product of a number and eight
Variable Let n represent the unknown number.
Expression $8n - 2$

Exercises

Translate each phrase into an algebraic expression.

1. eight inches taller than Mycala's height
2. twelve more than four times a number
3. the difference of sixty and a number
4. three times the number of tickets sold
5. fifteen dollars more than a saved amount
6. the quotient of the number of chairs and four
7. a number of books less than twenty-three
8. five more than six times a number
9. seven more boys than girls
10. twenty dollars divided among a number of friends minus three

1-2 Study Guide and Intervention *(continued)*

Variables and Expressions

Evaluate Expressions To evaluate an algebraic expression, replace the variable(s) with known values and follow the order of operations.

Substitution Property of Equality

Words If two quantities are equal, then one quantity can be replaced by the other.

Symbols For all numbers a and b, if $a = b$, then a may be replaced by b.

Example **ALGEBRA Evaluate each expression if $r = 6$ and $s = 2$.**

a. $\mathbf{8s - 2r}$

$8s - 2r = 8(2) - 2(6)$ Replace r with 6 and s with 2.

$= 16 - 12$ or 4 Multiply. Then subtract.

b. $\mathbf{3(r + s)}$

$3(r + s) = 3(2 + 6)$ Replace r with 6 and s with 2.

$= 3 \cdot 8$ or 24 Evaluate the parentheses. Then multiply.

c. $\mathbf{\frac{5rs}{4}}$

$\frac{5rs}{4} = 5rs \div 4$ Rewrite as a division expression.

$= 5(6)(2) \div 4$ Replace r with 6 and s with 2.

$= 60 \div 4$ or 15 Multiply. Then divide.

Exercises

ALGEBRA Evaluate each expression if $x = 10$, $y = 5$, and $z = 1$.

1. $x + y - z$ **2.** $\frac{x}{y}$ **3.** $2x + 4z$ **4.** $xy + z$

5. $\frac{6y}{10z}$ **6.** $x(2 + z)$ **7.** $x - 2y$ **8.** $\frac{(x + y)}{z}$

ALGEBRA Evaluate each expression if $r = 2$, $s = 3$, and $t = 12$.

9. $2t - rs$ **10.** $\frac{t}{rs}$ **11.** $t(4 + r)$ **12.** $4s + 5r$

13. $\frac{5t}{(r + 3)}$ **14.** $(t - 2s)7$ **15.** $\frac{10t}{4s}$ **16.** $(t + r) - (r + s)$

1-3 Study Guide and Intervention

Properties

Properties of Addition and Multiplication In algebra, there are certain statements called **properties** that are true for any numbers.

Property	Explanations	Example
Commutative Property of Addition	$a + b = b + a$	$6 + 3 = 3 + 6$ $9 = 9$
Commutative Property of Multiplication	$a \cdot b = b \cdot a$	$4 \cdot 5 = 5 \cdot 4$ $20 = 20$
Associative Property of Addition	$(a + b) + c =$ $a + (b + c)$	$(3 + 4) + 7 = 3 + (4 + 7)$ $14 = 14$
Associative Property of Multiplication	$(a \cdot b) \cdot c =$ $a \cdot (b \cdot c)$	$(2 \cdot 5) \cdot 8 = 2 \cdot (5 \cdot 8)$ $80 = 80$
Additive Identity	$a + 0 = 0 + a = a$	$10 + 0 = 0 + 10 = 10$
Multiplicative Identity	$a \cdot 1 = 1 \cdot a = a$	$5 \cdot 1 = 1 \cdot 5 = 5$
Multiplicative Property of Zero	$a \cdot 0 = 0 \cdot a = 0$	$15 \cdot 0 = 0 \cdot 15 = 0$

Example 1 **Is subtraction of whole numbers associative? If not, give a counterexample.**

$(9 - 4) - 2 \stackrel{?}{=} 9 - (4 - 2)$ State the conjecture.

$5 - 2 \stackrel{?}{=} 9 - 2$ Simplify.

$3 \stackrel{?}{=} 7$ Simplify.

This is a counterexample. So, subtraction of whole numbers is not associative.

Example 2 **Name the property shown by the statement.**

$15 \times b = b \times 15$ The order of the numbers and variables changed. This is the Commutative Property of Multiplication.

Exercises

1. State whether the following conjecture is true or false: The multiplicative identity applies to division also. If false, give a counterexample.

Name the property shown by each statement.

2. $75 + 25 = 25 + 75$

3. $2 \cdot (3 \cdot 4) = (2 \cdot 3) \cdot 4$

4. $14 \cdot 1 = 14$

5. $p \cdot 0 = 0$

1-3 Study Guide and Intervention *(continued)*

Properties

Simplify Algebraic Expressions To **simplify** an algebraic expression, perform all possible operations. Properties can be used to help simplify an expression that contains variables.

Example **Simplify each expression.**

a. $\mathbf{(9 + r) + 7}$

$(9 + r) + 7 = (r + 9) + 7$	Commutative Property of Addition
$= r + (9 + 7)$	Associative Property of Addition
$= r + 16$	Add 9 and 7.

b. $\mathbf{3 \cdot (x \cdot 5)}$

$3 \cdot (x \cdot 5) = 3 \cdot (5 \cdot x)$	Commutative Property of Multiplication
$= (3 \cdot 5) \cdot x$	Associative Property of Multiplication
$= 15x$	Multiply 3 and 5.

Exercises

Simplify each expression.

1. $24 + (x + 6)$

2. $3 \cdot (4a)$

3. $9 + (12 + c)$

4. $13d \cdot 0$

5. $(3 + f) + 17$

6. $11 + (m + 5)$

7. $(b + 0) + 7$

8. $15(a \cdot 1)$

9. $4w(6)$

10. $(n + 7) + 12$

11. $(7 \cdot x) \cdot 8$

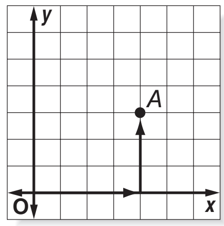
12. $21 \cdot (s \cdot 0)$

1-4 Study Guide and Intervention

Ordered Pairs and Relations

Ordered Pairs In mathematics, a **coordinate system** is used to locate points. The horizontal number line is called the ***x*-axis** and the vertical number line is called the ***y*-axis**. The point where the two axes intersect is the **origin** (0, 0). An **ordered pair** of numbers is used to locate points in the coordinate plane. The point (4, 3) has an ***x*-coordinate** of 4 and a ***y*-coordinate** of 3.

Example 1 **Graph *A*(4, 3) on the coordinate plane.**

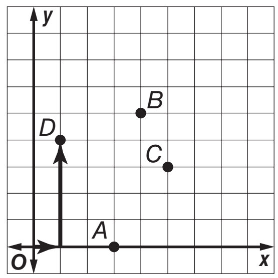


Step 1 Start at the origin.

Step 2 Since the *x*-coordinate is 4, move 4 units to the right.

Step 3 Since the *y*-coordinate is 3, move 3 units up. Draw a dot.

Example 2 **Write the ordered pair that names point *D*.**



Step 1 Start at the origin.

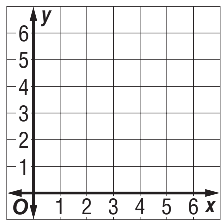
Step 2 Move right on the *x*-axis to find the *x*-coordinate of point *D*, which is 1.

Step 3 Move up the *y*-axis to find the *y*-coordinate, which is 4.

The ordered pair for point *D* is (1, 4).

Exercises

Graph each ordered pair on the coordinate plane.

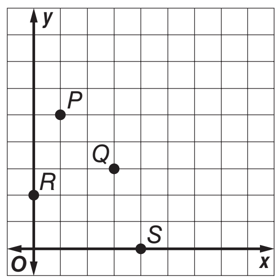


1. *A*(4, 1) **2.** *B*(2, 0)

3. *C*(1, 3) **4.** *D*(5, 2)

5. *E*(0, 3) **6.** *F*(6, 4)

Refer to the coordinate plane shown at the right. Write the ordered pair that names each point.



7. *P* **8.** *Q*

9. *R* **10.** *S*

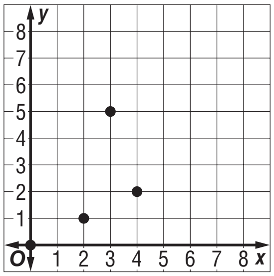
1-4 Study Guide and Intervention *(continued)*

Ordered Pairs and Relations

Relations A **relation** is a set of ordered pairs, such as {(0, 3), (1, 2), (3, 6), (7, 4)}. A relation can also be shown in a table or a graph. The set of *x*-coordinates is the **domain** of the relation, while the set of *y*-coordinates is the **range** of the relation.

Example **Express the relation {(0, 0), (2, 1), (4, 2), (3, 5)} as a table and as a graph. Then determine the domain and range.**

x	y
0	0
2	1
4	2
3	5



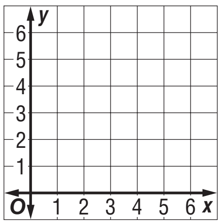
The domain is {0, 2, 4, 3}, and the range is {0, 1, 2, 5}.

Exercises

Express each relation as a table and as a graph. Then determine the domain and range.

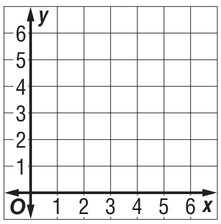
1. {(4, 6), (0, 3), (1, 4)}

x	y



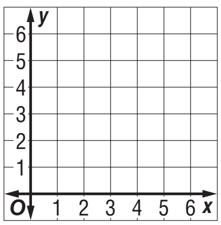
2. {(2, 5), (5, 3), (2, 2)}

x	y



3. {(1, 2), (3, 4), (5, 6)}

x	y



1-5 Study Guide and Intervention

Words, Equations, Tables, and Graphs

Represent Functions Functions are relations in which each member of the domain is paired with *exactly* one member in the range. The **function rule** describes the operation(s) which must be performed on a domain value to get the corresponding range value. **Function tables** organize and display the input values (the x-coordinates), the function rule, and the output values (the y-coordinates).

Example **TICKETS June is ordering tickets for a show. Tickets cost $22 each and there is a $6 surcharge per order. Make a function table for 4 different input values and write an algebraic expression for the rule. Then state the domain and range of the function.**

Step 1 Create a function table showing the input, rule, and output. Enter 4 different input values.

Step 2 The phrase "Tickets cost $22 each and there is a $6 surcharge per order" translates to $22x + 6$. Use the rule to complete the table.

Input (x)	Rule: $22x + 6$	Output (y)
1	$22(1) + 6$	28
2	$22(2) + 6$	50
3	$22(3) + 6$	72
4	$22(4) + 6$	94

Step 3 The domain is {1, 2, 3, 4}. The range is {28, 50, 72, 94}.

Exercises

For each ticket cost and surcharge given below, make a function table for 4 different input values and write an algebraic expression for the rule. Then state the domain and range of the function.

1. Ticket cost: $8; surcharge: $1.50

Input (x)	Rule:	Output (y)

2. Ticket cost: $12; surcharge: $3

Input (x)	Rule:	Output (y)

1-5 Study Guide and Intervention *(continued)*

Words, Equations, Tables, and Graphs

Multiple Representations Functions can be described as words, equations, tables and graphs.

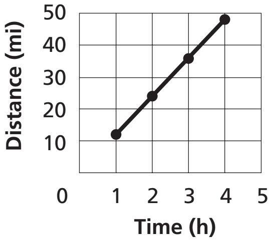
Words The distance biked is equal to 12 miles per hour times the number of hours.

Equation $d = 12t$

Table

Time (h)	Distance (mi)
1	12
2	24
3	36
4	48

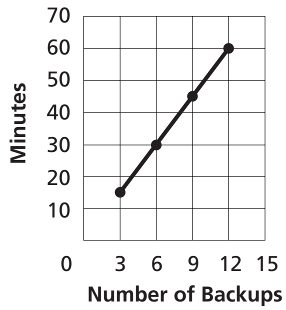
Graph



Example **FILE PROTECTION Tori's computer backs up the file she is working on every 5 minutes. Make a function table to find the time for 3, 6, 9, and 12 backups. Then graph the ordered pairs.**

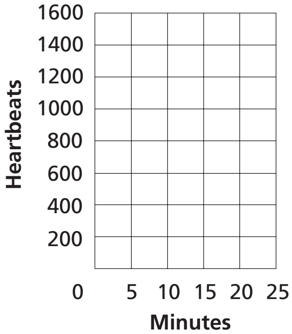
Let m represent the number of minutes and b represent the number of backups. So, the rule is $m = 5b$.

Input (x)	$5b$	Output (y)
3	5(3)	15
6	5(6)	30
9	5(9)	45
12	5(12)	60



Exercise

1. Viktor's heart beats 72 times a minute.
   a. **ALGEBRAIC** Write an equation to find the number of times Viktor's heart beats for any number of minutes.
   b. **TABULAR** Make a function table to find the number of times Viktor's heart beats in 5, 10, 15, and 20 minutes.
   c. **GRAPHICAL** Graph the ordered pairs for the function.



Input (x)		Output (y)

1-6 Study Guide and Intervention

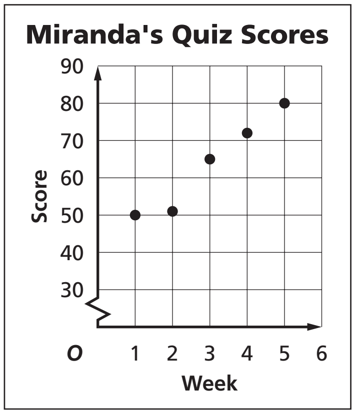
Scatter Plots

Construct Scatter Plots A **scatter plot** is a graph that shows the relationship between two sets of data. In a scatter plot, two sets of data are graphed as ordered pairs on a coordinate system.

Example **SCHOOL The table shows Miranda's math quiz scores for the last five weeks. Make a scatter plot of the data.**

Since the points are showing an upward trend from left to right, the data suggest a positive relationship.

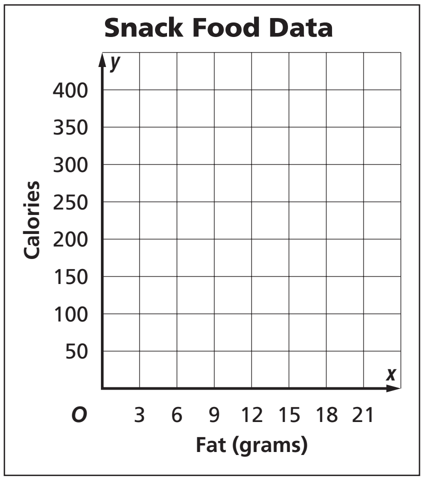
Week	Score
1	50
2	51
3	65
4	72
5	80



Exercise

FOOD The table below shows the fat grams and calories for several snack foods.

Food	Fat grams per serving	Calories per serving
doughnut	13	306
corn chips	13	200
pudding	3	150
cake	13	230
snack crackers	6	140
ice cream (light)	5	130
yogurt	2	70
cheese pizza	18	410

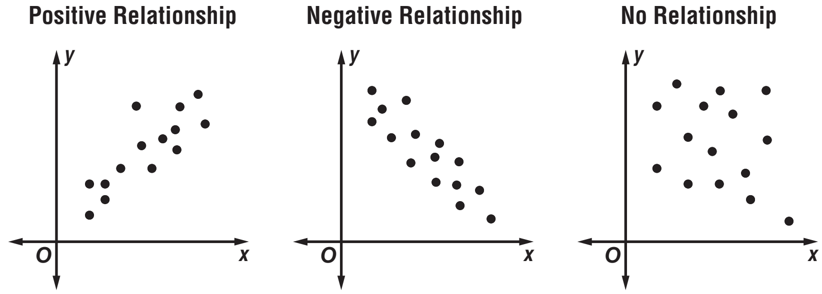


1. Make a scatter plot of the data in the table.

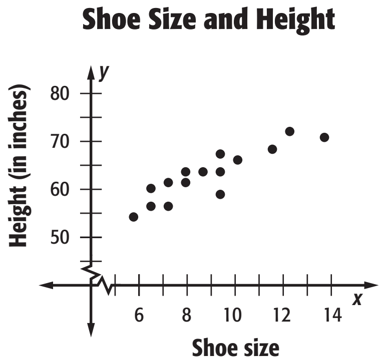
1-6 Study Guide and Intervention *(continued)*

Scatter Plots

Analyze Scatter Plots A scatter plot may show a pattern or relationship of the data.



Example **SHOE SIZE AND HEIGHT Determine whether a scatter plot of shoe size and height of people at a gym might show a *positive, negative,* or *no* relationship. Explain your answer.**



Height affects shoe size. A person's shoe size increases as their height increases. Therefore, a scatter plot of the data would show a positive relationship.

Exercises

Determine whether a scatter plot of the data for the following might show a *positive, negative,* or *no* relationship. Explain your answer.

1. fat grams and the amount of calories in food

2. time spent relaxing and blood pressure levels

3. age of a child and number of siblings

4. age of a tree and its height

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2-1 Study Guide and Intervention

Integers and Absolute Value

Compare and Order Integers The set of **integers** can be written {…, −3, −2, −1, 0, 1, 2, 3, …} where … means *continues indefinitely*. Two integers can be compared using an **inequality**, which is a mathematical sentence containing < or >.

Example 1 **Write an integer for each situation.**

a. 16 feet below the surface

The integer is −16.

b. 5 strokes over par

The integer is +5 or 5.

Example 2 **Use the integers graphed on the number line below.**

Replace each ● with < or > to make a true sentence.

a. **−6 ● −2**

−2 is greater since it lies to the right of −6. So write −6 < −2.

b. **3 ● −4**

3 is greater since it lies to the right of −4. So write 3 > −4.

Exercises

Write an integer for each situation.

1. 2 inches less than normal

2. 13°F above average

3. a deposit of $50

4. a loss of 8 yards

Replace each ● with <, >, or = to make a true sentence.

5. 4 ● −4

6. 8 ● 12

7. −7 ● −5

8. 2 ● 5

9. −1 ● 1

10. 4 ● −3

11. 6 ● 8

12. −2 ● 12

13. 9 ● −1

14. −6 ● −6

15. 5 ● −3

16. −10 ● 2

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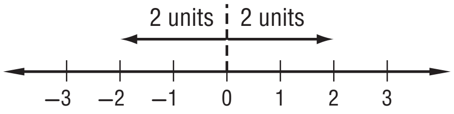
2-1 Study Guide and Intervention *(continued)*

Integers and Absolute Value

Absolute Value Numbers on opposite sides of zero and the same distance from zero have the same **absolute value**.



The symbol for absolute value is two vertical bars on either side of the number. $|2| = 2$ and $|-2| = 2$

Example 1 **Evaluate each expression.**

a. $|-4|$

$|-4| = 4$ On the number line, −4 is 4 units from 0.

b. $|-3| + |6|$

$$|-3| + |6| = 3 + 6 \quad |-3| = 3, |6| = 6$$
$$= 9 \quad \text{Simplify.}$$

Example 2 **Evaluate $|x| - 7$ if $x = -8$.**

$|x| - 7 = |-8| - 7$ Replace x with −8.

$= 8 - 7$ The absolute value of −8 is 8.

$= 1$ Simplify.

Exercises

Evaluate each expression.

1. $|-6|$ **2.** $|15|$ **3.** $|-12|$ **4.** $|21|$

5. $|4| - |2|$ **6.** $|-8| + |-3|$ **7.** $|-10| - |-6|$ **8.** $|12| + |-4|$

ALGEBRA Evaluate each expression if $x = 8$ and $y = -3$.

9. $12 + |y|$ **10.** $x - |y|$ **11.** $2|x| + 3|y|$

12. $x + |y|$ **13.** $6|y|$ **14.** $3x - 4|y|$

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2-2 Study Guide and Intervention

Adding Integers

Adding Integers with the Same Sign	Add their absolute values. The sum is: • positive if both integers are positive. • negative if both integers are negative.

Example 1 **Find the sum $-3 + (-4)$.**

$-3 + (-4) = -7$ Add $|-3|$ and $|-4|$. The sum is negative.

Adding Integers with Different Signs	Subtract their absolute values. The sum is: • positive if the positive integer's absolute value is greater. • negative if the negative integer's absolute value is greater.

Example 2 **Find each sum.**

a. $-5 + 4$

$-5 + 4 = |-5| - |4|$ Subtract $|4|$ from $|-5|$.
$= 5 - 4$ or 1 Simplify.
$= -1$ The sum is negative because $|-5| > |4|$.

b. $6 + (-2)$

$6 + (-2) = |6| - |-2|$ Subtract $|-2|$ from $|6|$.
$= 6 - 2$ or 4 Simplify.
$= 4$ The sum is positive because $|6| > |-2|$.

Exercises

Find each sum.

1. $6 + (-3)$
2. $-3 + (-5)$
3. $7 + (-3)$
4. $-4 + (-4)$
5. $-8 + 5$
6. $-12 + (-10)$
7. $6 + (-13)$
8. $-14 + 4$
9. $6 + (-6)$
10. $-15 + (-5)$
11. $-9 + 8$
12. $20 + (-8)$
13. $-19 + (-11)$
14. $17 + (-9)$
15. $-16 + (-5)$
16. $-12 + 14$
17. $9 + (-25)$
18. $-36 + 19$
19. $7 + (-18)$
20. $-12 + (-15)$
21. $10 + (-14)$
22. $-33 + 19$
23. $-20 + (-5)$
24. $-12 + (-10)$
25. $-15 + 4$
26. $-34 + 29$
27. $46 + (-32)$

2-2 Study Guide and Intervention *(continued)*

Adding Integers

Add More Than Two Integers Two numbers with the same absolute value but different signs are **opposites**. An integer and its opposite are also called **additive inverses**. This property is useful when adding 2 or more integers.

Additive Inverse Property

Words	The sum of any number and its additive inverse is zero.
Example	$5 + (-5) = 0$
Symbols	$a + (-a) = 0$

Example **Find each sum.**

a. –7 + (–16) + 7

$-7 + (-16) + 7 = -7 + 7 + (-16)$	Commutative Property
$= 0 + (-16)$	Additive Inverse Property
$= -16$	Identity Property of Addition

b. 12 + (–4) + 9 + (–7)

$12 + (-4) + 9 + (-7) = 12 + 9 + (-4) + (-7)$	Commutative Property
$= (12 + 9) + [-4 + (-7)]$	Associative Property
$= 21 + (-11)$ or 10	Simplify.

Exercises

Find each sum.

1. $2 + 14 + (-2)$

2. $-8 + (-7) + 8$

3. $-13 + 11 + (-4)$

4. $7 + (-5) + (-6)$

5. $15 + 14 + (-12)$

6. $-9 + 17 + (-3)$

7. $24 + (-5) + 3$

8. $54 + 39 + (-54)$

9. $-42 + 20 + (-8)$

10. $-11 + (-6) + 22$

11. $35 + (-43) + (-4)$

12. $-100 + 50 + (-25)$

13. $6 + (-14) + (-5) + (-6)$

14. $-18 + 9 + (-7) + 18$

15. $5 + 13 + (-11) + 6$

16. $-20 + 15 + (-10) + 3$

17. $-33 + (-7) + 20 + 9$

18. $16 + (-12) + 21 + (-25)$

2-3 Study Guide and Intervention

Subtracting Integers

Subtracting Integers	To subtract an integer, add its additive inverse.

Example 1 **Find each difference.**

a. 9 − 17

$9 - 17 = 9 + (-17)$ To subtract 17, add −17.

$= -8$ Simplify.

b. −7 − 3

$-7 - 3 = -7 + (-3)$ To subtract 3, add −3.

$= -10$ Simplify.

Example 2 **Find each difference.**

a. 4 − (−5)

$4 - (-5) = 4 + 5$ To subtract −5, add +5.

$= 9$ Simplify.

b. −6 − (−2)

$-6 - (-2) = -6 + 2$ To subtract −2, add +2.

$= -4$ Simplify.

Exercises

Find each difference.

1. $9 - 16$

2. $7 - 19$

3. $12 - 21$

4. $-5 - 3$

5. $-8 - 9$

6. $-13 - 17$

7. $7 - (-4)$

8. $9 - (-9)$

9. $-11 - (-2)$

10. $-6 - (-9)$

11. $-6 - 4$

12. $-16 - (-20)$

13. $-14 - 4$

14. $8 - (-6)$

15. $-10 - (-6)$

16. $13 - (-17)$

17. $24 - (-16)$

18. $17 - (-9)$

19. $-24 - 8$

20. $18 - (-9)$

21. $26 - 49$

22. $-45 - (-26)$

23. $-15 - (-25)$

24. $29 - (-6)$

2-3 Study Guide and Intervention *(continued)*

Subtracting Integers

Evaluate Expressions Use the rule for subtracting integers to evaluate expressions.

Example **Evaluate each expression.**

a. $x - 16$ if $x = 6$.

$x - 16 = 6 - 16$	Write the expression. Replace *x* with 6.
$= 6 + (-16)$	To subtract 16, add its additive inverse, −16.
$= -10$	Add 6 and −16.

b. $a - b - c$ if $a = 7$, $b = 2$, and $c = -3$.

$a - b - c = 7 - 2 - (-3)$	Replace *a* with 7, *b* with 2, and *c* with −3.
$= 5 - (-3)$	Use order of operations.
$= 5 + 3$	To subtract −3, add its additive inverse, 3.
$= 8$	Add 5 and 3.

Exercises

ALGEBRA Evaluate each expression if $a = 11$, $b = -1$, and $c = -8$.

1. $a - 14$

2. $b - 5$

3. $12 - c$

4. $33 - a$

5. $c - 8$

6. $-19 - b$

7. $-5 - c$

8. $3 - a$

9. $b - (-1)$

10. $a - (-7)$

11. $6 - b$

12. $c - (-12)$

13. $a - b$

14. $a - c$

15. $c - b$

16. $b - c$

17. $c - a$

18. $b - a$

19. $a - b - c$

20. $a + b - c$

21. $b - c - a$

22. $c - a + b$

23. $b - (-a) - c$

24. $c + b - a$

2-4 Study Guide and Intervention

Multiplying Integers

Multiplying Integers with Different Signs	The product of two integers with different signs is negative.

Example 1 **Find each product.**

a. $4(-3)$

$4(-3) = -12$

b. $-8(5)$

$-8(5) = -40$

Multiplying Integers with the Same Sign	The product of two integers with the same sign is positive.

Example 2 **Find each product.**

a. $6(6)$

$6(6) = 36$

b. $-7(-4)$

$-7(-4) = 28$

Example 3 **Find $6(-3)(-2)$.**

$6(-3)(-2) = [6(-3)](-2)$ Use the Associative Property.

$= -18(-2)$ $6(-3) = -18$

$= 36$ $-18(-2) = 36$

Exercises

Find each product.

1. $-5(7)$

2. $6(-9)$

3. $-10 \cdot 4$

4. $-12 \cdot -2$

5. $5(-11)$

6. $-15(-4)$

7. $-14(2)$

8. $6(14)$

9. $-18 \cdot 2$

10. $-9(10)$

11. $12(-6)$

12. $-11(-11)$

13. $-4(-4)(5)$

14. $6(-7)(2)$

15. $-10(-4)(-6)$

16. $-7(-3)(2)$

17. $-9(4)(2)$

18. $6(-4)(-12)$

19. $11(3)(-2)$

20. $-5(-6)(7)$

21. $-3(-4)(-8)$

22. $22(3)(-3)$

23. $-8(10)(-2)$

24. $-6(5)(-9)$

2-4 Study Guide and Intervention *(continued)*

Multiplying Integers

Algebraic Expressions Use the rules for multiplying integers to simplify and evaluate algebraic expressions.

Example 1 **Simplify $-3a(-12b)$.**

$-3a(-12b) = (-3)(a)(-12)(b)$	$-3a = (-3)(a)$, $-12b = (-12)(b)$
$= (-3 \cdot -12)(a \cdot b)$	Commutative Property of Multiplication
$= 36ab$	$-3 \cdot -12 = 36$, $a \cdot b = ab$

Example 2 **Evaluate $4xy$ if $x = 3$ and $y = -5$.**

$4xy = 4(3)(-5)$	Replace x with 3, and y with -5.
$= [4(3)](-5)$	Associative Property of Multiplication
$= 12(-5)$	The product of 4 and 3 is positive.
$= -60$	The product of 12 and -5 is negative.

Exercises

ALGEBRA Simplify each expression.

1. $9(-3w)$

2. $2e \cdot 9f$

3. $-8 \cdot 7m$

4. $-4s(-7)$

5. $10p(-5q)$

6. $n \cdot 6 \cdot 8$

7. $-3a(15b)$

8. $-9x \cdot (-4y)$

9. $-c \cdot 11d$

ALGEBRA Evaluate each expression if $x = -4$ and $y = 8$.

10. $4x$

11. $3y$

12. $-12x$

13. $-6y$

14. xy

15. $-xy$

16. $-2xy$

17. $5xy$

18. $-3x(-y)$

2-5 Study Guide and Intervention

Dividing Integers

Dividing Integers with the Same Sign	The quotient of two integers with the same sign is positive.

Example 1 **Find each quotient.**

a. $\mathbf{14 \div 2}$

$14 \div 2 = 7$ — The dividend and the divisor have the same sign. The quotient is positive.

b. $\mathbf{\frac{-25}{-5}}$

$\frac{-25}{-5} = -25 \div (-5)$ — The dividend and divisor have the same sign.

$= 5$ — The quotient is positive.

Dividing Integers with Different Signs	The quotient of two integers with different signs is negative.

Example 2 **Find each quotient.**

a. $\mathbf{36 \div (-4)}$ — The signs are different.

$36 \div (-4) = -9$ — The quotient is negative.

b. $\mathbf{-\frac{42}{6}}$ — The signs are different.

$-\frac{42}{6} = -7$ — The quotient is negative.

Exercises

Find each quotient.

1. $32 \div (-4)$ **2.** $-18 \div (-2)$ **3.** $-24 \div 6$

4. $-36 \div (-2)$ **5.** $50 \div (-5)$ **6.** $-81 \div (-9)$

7. $-72 \div (-2)$ **8.** $-45 \div 3$ **9.** $-60 \div (-12)$

10. $99 \div (-11)$ **11.** $-200 \div (-4)$ **12.** $38 \div (-2)$

13. $-144 \div 12$ **14.** $100 \div (-5)$ **15.** $-200 \div (-20)$

16. $\frac{-28}{2}$ **17.** $\frac{36}{-4}$ **18.** $\frac{-150}{-25}$

2-5 Study Guide and Intervention *(continued)*

Dividing Integers

Mean (Average) To find the **mean,** or average, of a set of numbers, find the sum of the numbers and then divide by the number of items in the set. Use the rules for dividing integers to find the mean.

Example **OCEANOGRAPHY The diving depths in feet of 7 scuba divers studying schools of fish were –12, –9, –15, –8, –20, –17, and –10. Find the mean diving depth.**

$$\frac{-12 + (-9) + (-15) + (-8) + (-20) + (-17) + (-10)}{7} = \frac{-91}{7}$$

Find the sum of the diving depths.
Divide by the number of divers.

$$= -13$$

Simplify.

The mean diving depth is –13 feet, or 13 feet below sea level.

Exercises

1. **WEATHER** The low temperatures in degrees Fahrenheit for a week were –3, 5, –9, 2, 6, –11, and –4. Find the mean temperature.

2. **MONEY** The last 6 entries in Ms. Caudle's checkbook ledger show both deposits and withdrawals. Ms. Caudle wrote down \$100, –\$20, –\$35, \$250, –\$150, and –\$85. What is the mean dollar amount for these entries?

3. **GOLF** During 5 rounds of golf, James had scores of 2, –1, 0, –2, and –4. Find the mean of his golf scores.

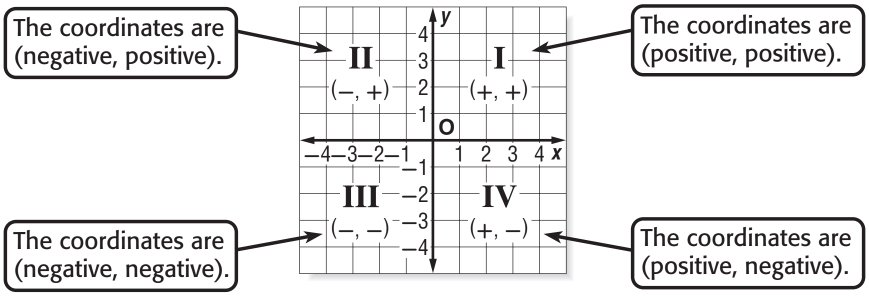
4. **TRAINING** To train himself for a motivation, josh runs every day. Last week he ran 3 miles, 7 miles, 3 miles, 4 miles, 7 miles, 10 miles and 5 miles. What is the mean number of miles he ran last week?

5. **ROCK CLIMBING** A rock climber makes several changes in position while attempting to scale a cliff face. She ascends 15 feet, descends 7 feet, ascends 22 feet, descends 13 feet, and then ascends another 28 feet. What is her mean change in position?

NAME ________________________________ DATE ____________ PERIOD ____________

2-6 Study Guide and Intervention

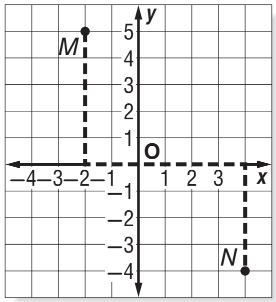
Graphing in Four Quadrants



Example **Graph and label each point on a coordinate plane. Name the quadrant in which each point lies.**

a. ***M*(–2, 5)**
Start at the origin. Move 2 units left.
Then move 5 units up and draw a dot.
Point *M*(–2, 5) is in Quadrant II.

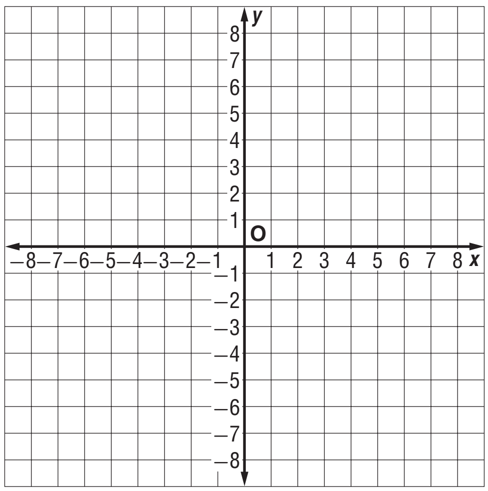
b. ***N*(4, –4)**
Start at the origin. Move 4 units right.
Then move 4 units down and draw a dot.
Point *N*(4, –4) is in Quadrant IV.



Exercises

Graph and label each point on the coordinate plane. Name the quadrant in which each point is located.

1. *A*(2, 6)
2. *B*(–1, 4)
3. *C*(0, –5)
4. *D*(–4, –3)
5. *E*(2, 0)
6. *F*(3, –2)
7. *G*(–4, 4)
8. *H*(2, –5)
9. *I*(6, 3)
10. *J*(–5, –8)
11. *K*(3, –5)
12. *L*(–7, –3)



2-6 Study Guide and Intervention *(continued)*

Graphing in Four Quadrants

Graph Algebraic Relationships A coordinate graph can be used to show relationships between two numbers.

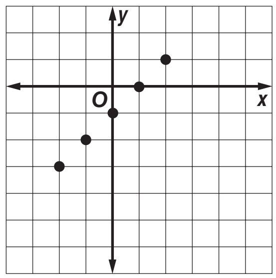
Example **MONEY The difference between Zora's and Charlie's bank accounts is $1. If x represents Zora's bank account and y represents Charlie's bank account, make a function table of possible values for x and y. Graph the ordered pairs and describe the graph.**

Step 1 Make a table. Choose values for x and y that have a difference of 1.

Step 2 Graph the ordered pairs.

The points are along a diagonal line that crosses the x-axis at $x = 1$.

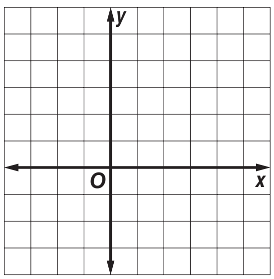
$x - y = 1$		
x	y	(x, y)
2	1	(2, 1)
1	0	(1, 0)
0	–1	(0, –1)
–1	–2	(–1, –2)
–2	–3	(–2, –3)



Exercises

1. **TEMPERATURE** The sum of two temperatures is 3°F. If x represents the first temperature and y represents the second temperature, make a function table of possible values for x and y. Graph the ordered pairs and describe the graph.

$x + y = 3$		
x	y	(x, y)



2-7 Study Guide and Intervention

Translations and Reflections on the Coordinate Plane

Transformations A **transformation** is an operation that maps an original geometric figure onto a new figure called the **image.** A **translation** and a **reflection** are two types of transformations on the coordinate plane.

Translation

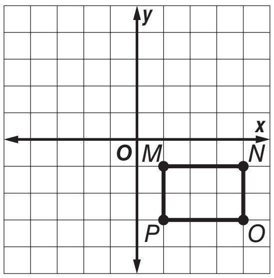
- called a "slide"
- image is the same shape and the same size as original figure
- orientation is the same as the original figure

Reflection

- called a "flip"
- figures are mirror images of each other
- image is the same shape and same size as original figure
- orientation is *different* from the original figure

An ordered pair (a, b) can be used to describe a translation, where every point $P(x, y)$ is moved to an image $P'(x + a, y + b)$.

Example **Rectangle *MNOP* is shown at the right. If it is translated 4 units to the left and 5 units up, find the coordinates of the vertices of the image.**

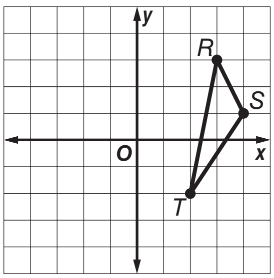


This translation can be written as (−4, 5). To find the coordinates of the translated image, add −4 to each *x*-coordinate and add 5 to each *y*-coordinate.

vertex		translation		Image
$M(1, -1)$	+	$(-4, 5)$	→	$M'(-3, 4)$
$N(4, -1)$	+	$(-4, 5)$	→	$N'(0, 4)$
$O(4, -3)$	+	$(-4, 5)$	→	$O'(0, 2)$
$P(1, -3)$	+	$(-4, 5)$	→	$P'(-3, 2)$

Exercises

1. Triangle *RST* is shown on the coordinate plane. Find the coordinates of the vertices of the image if triangle *RST* is translated 6 units to the left and 3 units down.

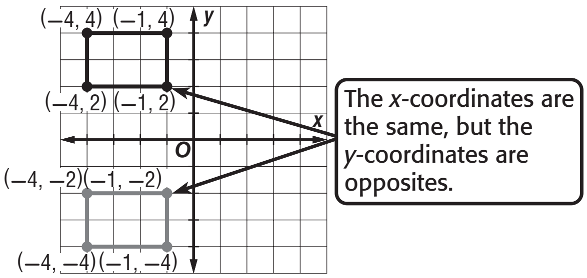


2-7 Study Guide and Intervention *(continued)*

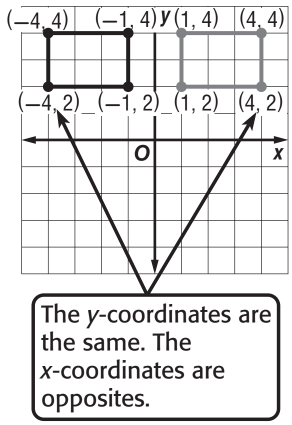
Translations and Reflections on the Coordinate Plane

Graph Transformations When reflecting a figure, every point of the original figure has a corresponding point on the other side of the **line of symmetry.** Corresponding points are the same distance from the line of symmetry.

Reflection over the *x*-axis

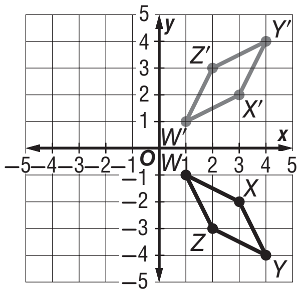


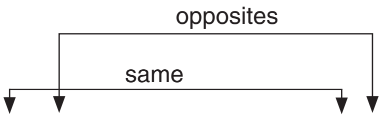
Reflection over the *y*-axis



Example **The vertices of figure *WXYZ* are *W*(1, –1), *X*(3, –2), *Y*(4, –4), and *Z*(2, –3). Graph the figure and its image after a reflection over the *x*-axis.**

To find the coordinates of the vertices of the image after a reflection over the *x*-axis, use the same *x*-coordinate. Replace the *y*-coordinate with its opposite.

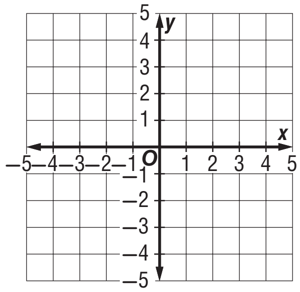




W(1, –1)	→	*W*′(1, 1)
X(3, –2)	→	*X*′(3, 2)
Y(4, –4)	→	*Y*′(4, 4)
Z(2, –3)	→	*Z*′(2, 3)

Exercises

1. The vertices of figure *JKLM* are *J*(–4, –2), *K*(–2, –2), *L*(–1, –4), and *M*(–5, –4). Graph the figure and its image after a reflection over the *y*-axis.



3-1 Study Guide and Intervention

Fractions and Decimals

Write Fractions as Decimals Some fractions, such as $\frac{1}{4}$ and $\frac{3}{5}$, can easily be written as decimals by making equivalent fractions with denominators of 10, 100, or 1,000.

All fractions can be written as decimals by dividing the numerator by the denominator. If the division ends or terminates with a remainder of 0, it is a **terminating decimal.** If the decimal number repeats without end it is a **repeating decimal.**

Example 1 **Write $\frac{7}{8}$ as a decimal.**

$\frac{7}{8}$

```
    0.875
 8)7.000
   6 4
     60
     56
      40
      40
       0
```

0.875 is a terminating decimal.

Example 2 **Write $\frac{4}{9}$ as a decimal.**

$\frac{4}{9}$

```
    0.444
 9)4.000
   3 6
     40
     36
      40
      36
       4
```

0.444... is a repeating decimal. You can indicate that a decimal repeats by writing a bar or line over the repeating digit(s): $\frac{4}{9} = 0.\overline{4}$.

Exercises

Write each fraction as a decimal. Use a bar to show a repeating decimal.

1. $\frac{7}{20}$ **2.** $\frac{2}{11}$ **3.** $\frac{5}{9}$

4. $\frac{5}{6}$ **5.** $\frac{6}{25}$ **6.** $\frac{5}{20}$

7. $\frac{3}{5}$ **8.** $\frac{7}{25}$ **9.** $\frac{4}{15}$

10. $\frac{12}{32}$ **11.** $\frac{9}{10}$ **12.** $\frac{5}{11}$

13. $-\frac{7}{9}$ **14.** $\frac{27}{40}$ **15.** $-\frac{2}{3}$

3-1 Study Guide and Intervention *(continued)*

Fractions and Decimals

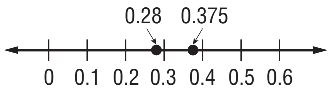
Compare Fractions and Decimals It may be easier to compare numbers when they are written as decimals.

Example 1 **Replace ● with <, >, or = to make 0.28 ● $\frac{3}{8}$ a true sentence.**

0.28 ● $\frac{3}{8}$

0.28 ● 0.375 Write $\frac{3}{8}$ as a decimal.

$0.28 < 0.375$ Compare the tenths place: $2 < 3$.

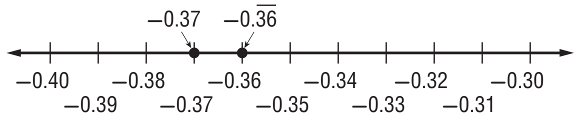


Example 2 **Replace ● with <, >, or = to make -0.37 ● $-\frac{4}{11}$ a true sentence.**

-0.37 ● $-\frac{4}{11}$

-0.37 ● $-0.\overline{36}$ Write $\frac{4}{11}$ as a decimal.

$-0.37 < -0.\overline{36}$ -0.37 is to the left of $-0.\overline{36}$ on the number line, so $-0.37 < -0.\overline{36}$.



Exercises

Replace each ● with <, >, or = to make a true sentence.

1. $\frac{5}{8}$ ● $\frac{6}{9}$

2. $\frac{4}{5}$ ● 0.8

3. $\frac{7}{8}$ ● $\frac{4}{5}$

4. 0.09 ● $\frac{1}{2}$

5. 0.3 ● $\frac{1}{3}$

6. $\frac{5}{12}$ ● $\frac{16}{40}$

7. $\frac{14}{27}$ ● 0.6

8. $-\frac{3}{10}$ ● $-\frac{2}{5}$

9. $\frac{3}{4}$ ● 0.75

10. 0.03 ● $\frac{4}{15}$

11. $\frac{13}{30}$ ● $\frac{5}{9}$

12. -0.55 ● $-\frac{7}{12}$

13. 0.16 ● $\frac{4}{25}$

14. $-\frac{11}{40}$ ● -0.02

15. $\frac{7}{8}$ ● 0.88

3-2 Study Guide and Intervention

Rational Numbers

Write Rational Numbers as Fractions A number that can be written as a fraction is called a **rational number.** Mixed numbers, integers, terminating decimals, and repeating decimals can all be written as fractions. Any number that can be expressed as $\frac{a}{b}$, where a and b are integers and $b \neq 0$ is a rational number.

Example **Write each number as a fraction.**

a. $3\frac{2}{5}$

$3\frac{2}{5} = \frac{17}{5}$ — Write the mixed number as an improper fraction.

b. -7

$-7 = -\frac{7}{1}$ — The denominator is 1.

c. 0.14

0.14 is 14 hundredths.

$0.14 = \frac{14}{100}$ or $\frac{7}{50}$ — Simplify.

d. $0.\overline{5}$

$0.\overline{5} = 0.555...$

$N = 0.555...$ — Let N represent the number.

$10N = 5.555...$ — Multiply each side by 10 because one digit repeats.

$$\begin{aligned} 10N &= 5.555... \\ -(N &= 0.555...) \\ \hline 9N &= 5 \end{aligned}$$

Subtract N from $10N$.

$\frac{9N}{9} = \frac{5}{9}$ — Divide each side by 9.

$N = \frac{5}{9}$ — Simplify.

Exercises

Write each number as a fraction.

1. $1\frac{1}{5}$

2. -2

3. 0.7

4. 0.32

5. $-0.\overline{1}$

6. $0.\overline{49}$

7. 5.28

8. $-7\frac{5}{6}$

9. $0.\overline{68}$

10. -9.08

11. $-0.0\overline{6}$

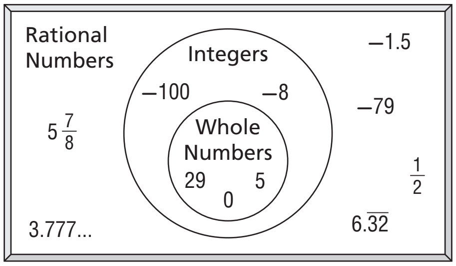
12. $6\frac{8}{11}$

3-2 Study Guide and Intervention *(continued)*

Rational Numbers

Identify and Classify Rational Numbers

Numbers can be classified into a variety of different sets. The diagram at the right illustrates the relationships among the sets of whole numbers, integers, and rational numbers.



Decimal numbers such as $\pi = 3.141592...$ and 6.767767776... are infinite and nonrepeating. They are called **irrational** numbers.

Example **Identify all sets to which each number belongs.**

a. −0.08 — This is neither a whole number nor an integer. Since −0.08 can be written as $-\frac{8}{100}$, it is rational.

b. 19 — This is a whole number, an integer, and a rational number.

c. 8.282282228... — This is a nonterminating and nonrepeating decimal. So, it is irrational.

d. −8 — This is an integer and a rational number.

Exercises

Identify all sets to which each number belongs.

1. −12

2. 8.5

3. 582

4. 0

5. −68

6. $\frac{1}{5}$

7. 8.98

8. 4.7829381...

9. 2,038

10. −1.45

11. $\frac{99}{5}$

12. $4.\overline{34}$

13. 9.09090909...

14. $-13\frac{1}{9}$

15. −739

3-3 Study Guide and Intervention

Multiplying Rational Numbers

Multiply Fractions To multiply fractions, multiply the numerators and multiply the denominators: $\frac{a}{b} \cdot \frac{c}{d} = \frac{a \cdot c}{b \cdot d}$, where $b, d \neq 0$. Fractions may be simplified either before or after multiplying. When multiplying negative fractions, assign the negative sign to the numerator.

Example **Find each product. Write in simplest form.**

a. $-\frac{8}{15} \cdot \frac{5}{7} = \frac{-8}{15} \cdot \frac{5}{7}$ Rewrite with the negative sign in the numerator.

$= \frac{-8}{\underset{3}{\cancel{15}}} \cdot \frac{\overset{1}{\cancel{5}}}{7}$ Simplify before multiplying by dividing 5 and 15 by their GCF, 5.

$= \frac{-8 \cdot 1}{3 \cdot 7}$ Multiply.

$= \frac{-8}{21} = -\frac{8}{21}$ Simplify.

b. $7\frac{1}{2} \cdot 2\frac{2}{3} = \frac{15}{2} \cdot \frac{8}{3}$ Rename mixed numbers as improper fractions.

$= \frac{\overset{5}{\cancel{15}}}{\underset{1}{\cancel{2}}} \cdot \frac{\overset{4}{\cancel{8}}}{\underset{1}{\cancel{3}}}$ Divide 15 and 3 by 3, and 8 and 2 by 2.

$= \frac{5 \cdot 4}{1 \cdot 1}$ Multiply.

$= \frac{20}{1}$ or 20 Simplify.

Exercises

Find each product. Write in simplest form.

1. $\frac{1}{2} \cdot \frac{3}{5}$

2. $-\frac{8}{9} \cdot \frac{5}{16}$

3. $\frac{4}{5} \cdot \frac{5}{8}$

4. $\frac{3}{10} \cdot \left(-\frac{1}{4}\right)$

5. $\frac{7}{9} \cdot \frac{11}{20}$

6. $\frac{2}{5} \cdot (-5)$

7. $-4\frac{4}{5} \cdot 1\frac{1}{6}$

8. $1\frac{5}{7} \cdot 10\frac{1}{2}$

9. $-2\frac{1}{8} \cdot \left(-4\frac{4}{7}\right)$

10. $2\frac{4}{9} \cdot \left(-3\frac{6}{11}\right)$

3-3 Study Guide and Intervention *(continued)*

Multiplying Rational Numbers

Evaluate Expressions With Fractions Algebraic expressions are expressions which contain one or more variables. Variables can represent fractions in algebraic expressions.

Example **Evaluate $\frac{2}{3}ab$ if $a = 3\frac{3}{7}$ and $b = -\frac{5}{12}$. Write the product in simplest form.**

$\frac{2}{3}ab = \frac{2}{3}\left(3\frac{3}{7}\right)\left(-\frac{5}{12}\right)$ Replace a with $3\frac{3}{7}$ and b with $-\frac{5}{12}$.

$= \frac{2}{3}\left(\frac{24}{7}\right)\left(\frac{-5}{12}\right)$ Rename $3\frac{3}{7}$ as $\frac{24}{7}$.

$= \frac{2}{3}\left(\frac{\overset{2}{\cancel{24}}}{7}\right)\left(\frac{-5}{\underset{1}{\cancel{12}}}\right)$ The GCF of 24 and 12 is 12.

$= \frac{2 \cdot 2(-5)}{3 \cdot 7}$ Multiply.

$= \frac{-20}{21} = -\frac{20}{21}$ Simplify.

Exercises

Evaluate each expression if $x = \frac{7}{10}$, $y = -4\frac{2}{5}$, and $z = -\frac{4}{7}$. Write the product in simplest form.

1. xy **2.** yz **3.** xyz

4. $5y$ **5.** $-5xy$ **6.** $\frac{1}{2}y$

7. $2\frac{3}{10}z$ **8.** $-\frac{2}{3}x$ **9.** $x \cdot x$

10. $28z$ **11.** $-y$ **12.** $y \cdot y$

13. $5\frac{5}{6}xz$ **14.** $\frac{2}{5}(-x)$ **15.** $\frac{9}{10}y$

3-4 Study Guide and Intervention

Dividing Rational Numbers

Divide Fractions Two numbers whose product is 1 are called multiplicative inverses or reciprocals. For any fraction $\frac{a}{b}$, where $a, b \neq 0$, $\frac{b}{a}$ is the multiplicative inverse and $\frac{a}{b} \cdot \frac{b}{a} = 1$. This means that $\frac{2}{3}$ and $\frac{3}{2}$ are multiplicative inverses because $\frac{2}{3} \cdot \frac{3}{2} = 1$.

To divide by a fraction, multiply by its multiplicative inverse: $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c} = \frac{ad}{bc}$, where $b, c, d \neq 0$.

Example **Find each quotient. Write in simplest form.**

a. $\frac{3}{4} \div \frac{5}{8} = \frac{3}{4} \cdot \frac{8}{5}$ Multiply by the multiplicative inverse of $\frac{5}{8}$, $\frac{8}{5}$.

$= \frac{3}{\cancel{4}_1} \cdot \frac{\cancel{8}^2}{5}$ Divide 4 and 8 by their GCF, 4.

$= \frac{6}{5}$ or $1\frac{1}{5}$ Simplify.

b. $-6\frac{2}{5} \div 2\frac{1}{5} = \frac{-32}{5} \div \frac{11}{5}$ Rename mixed numbers as improper fractions.

$= \frac{-32}{5} \cdot \frac{5}{11}$ Multiply by the multiplicative inverse of $\frac{11}{5}$, $\frac{5}{11}$.

$= \frac{-32}{\cancel{5}_1} \cdot \frac{\cancel{5}^1}{11}$ Divide out common factors.

$= \frac{-32}{11}$ or $-2\frac{10}{11}$ Simplify.

Exercises

Find each quotient. Write in simplest form.

1. $\frac{5}{16} \div \frac{5}{8}$

2. $\frac{7}{9} \div \frac{2}{3}$

3. $\frac{16}{21} \div \left(-\frac{2}{7}\right)$

4. $-\frac{4}{5} \div \frac{3}{10}$

5. $1\frac{1}{4} \div 2\frac{3}{8}$

6. $-8\frac{4}{7} \div 2\frac{1}{7}$

7. $\frac{18}{21} \div 3$

8. $-4\frac{5}{8} \div \left(-3\frac{1}{3}\right)$

3-4 Study Guide and Intervention *(continued)*

Dividing Rational Numbers

Divide Algebraic Fractions Algebraic fractions are fractions which contain one or more variables. You can divide algebraic fractions just as you would divide numerical fractions.

Example **Find $\frac{4}{qrs} \div \frac{10}{qs}$. Write the quotient in simplest form.**

$\frac{4}{qrs} \div \frac{10}{qs} = \frac{4}{qrs} \cdot \frac{qs}{10}$ Multiply by the reciprocal of $\frac{10}{qs}$, $\frac{qs}{10}$.

$= \frac{\overset{2}{\cancel{4}}}{\cancel{qrs}} \cdot \frac{\overset{1}{\cancel{qs}}}{\underset{5}{\cancel{10}}}$ Divide out common factors.

$= \frac{2}{5r}$ Simplify.

Exercises

Find each quotient. Write in simplest form.

1. $\frac{2x}{y} \div \frac{3}{y}$

2. $\frac{c}{4d} \div \frac{3}{8d}$

3. $\frac{4a}{b} \div \frac{2ac}{b}$

4. $\frac{m}{9} \div \frac{mn^2}{3}$

5. $\frac{ab}{9} \div \frac{bc}{12}$

6. $\frac{2st}{q} \div \frac{4t}{q}$

7. $\frac{10z}{xy} \div \frac{2}{5xyz}$

8. $\frac{8g}{3hi} \div \frac{4g}{15i}$

9. $\frac{7p}{9qr} \div \frac{3p}{18q}$

10. $\frac{x}{yz} \div \frac{4x}{11z}$

11. $\frac{2d}{3ef} \div \frac{5}{6ef}$

12. $\frac{3x}{5wy} \div \frac{6x}{20yz}$

13. $\frac{4ab}{3c} \div \frac{6b}{4c}$

14. $\frac{14jk}{3l} \div \frac{4j}{9l}$

15. $\frac{6a}{11bc} \div \frac{a}{44b}$

16. $\frac{15yz}{6x} \div \frac{10z}{3x}$

17. $\frac{de}{20f} \div \frac{e}{2f}$

18. $\frac{6i}{5gh} \div \frac{8i}{3h}$

3-5 Study Guide and Intervention

Adding and Subtracting Like Fractions

Add Like Fractions To add fractions with the same denominators, called **like denominators,** add the numerators and write the sum over the denominator.

So, $\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$, where $c \neq 0$.

Example 1 **Find $\frac{5}{12} + \frac{9}{12}$. Write in simplest form.**

$\frac{5}{12} + \frac{9}{12} = \frac{5+9}{12}$ — The denominators are the same. Add the numerators.

$= \frac{14}{12}$ or $1\frac{2}{12}$ or $1\frac{1}{6}$ — Simplify and rename to a mixed number.

Example 2 **Find $\frac{3}{8} + \left(-\frac{7}{8}\right)$. Write in simplest form.**

$\frac{3}{8} + \left(-\frac{7}{8}\right) = \frac{3+(-7)}{8}$ — The denominators are the same. Add the numerators.

$= \frac{-4}{8}$ or $-\frac{1}{2}$ — Simplify.

Example 3 **Find $1\frac{2}{9} + 3\frac{4}{9}$. Write in simplest form.**

$1\frac{2}{9} + 3\frac{4}{9} = (1 + 3) + \left(\frac{2}{9} + \frac{4}{9}\right)$ — Add the whole numbers and fractions separately or write as improper fractions.

$= 4 + \frac{2+4}{9}$ — Add the numerators.

$= 4\frac{6}{9}$ or $4\frac{2}{3}$ — Simplify.

Exercises

Find each sum. Write in simplest form.

1. $\frac{11}{12} + \frac{9}{12}$

2. $\frac{13}{15} + \frac{9}{15}$

3. $\frac{4}{9} + \frac{8}{9}$

4. $\frac{4}{20} + \left(-\frac{9}{20}\right)$

5. $\frac{5}{6} + \frac{5}{6}$

6. $-\frac{9}{10} + \frac{4}{10}$

7. $\frac{19}{20} - \frac{17}{20}$

8. $9 + 4\frac{3}{7}$

9. $7\frac{3}{4} + 3\frac{1}{4}$

10. $-6\frac{7}{12} + \left(-8\frac{11}{12}\right)$

11. $-4\frac{9}{14} + 3\frac{5}{14}$

12. $2\frac{3}{5} + \left(-\frac{1}{5}\right)$

3-5 Study Guide and Intervention *(continued)*

Adding and Subtracting Like Fractions

Subtract Like Fractions To subtract fractions with like denominators, subtract the numerators and write the difference over the denominator. So, $\frac{a}{c} - \frac{b}{c} = \frac{a-b}{c}$, where $c \neq 0$.

Example 1 **Find $\frac{3}{8} - \frac{5}{8}$. Write in simplest form.**

$\frac{3}{8} - \frac{5}{8} = \frac{3-5}{8}$ The denominators are the same. Subtract the numerators.

$= -\frac{2}{8}$ or $-\frac{1}{4}$ Simplify.

Example 2 **Evaluate $x - y$ when $x = 7\frac{1}{3}$ and $y = 5\frac{2}{3}$. Write in simplest form.**

$x - y = 7\frac{1}{3} - 5\frac{2}{3}$ Replace x with $7\frac{1}{3}$ and y with $5\frac{2}{3}$.

$= 7\frac{1}{3} - 5\frac{2}{3} = 6\frac{4}{3} - 5\frac{2}{3}$ Since $\frac{1}{3} < \frac{2}{3}$, think of $7\frac{1}{3}$ as $6\frac{3}{3} + \frac{1}{3}$, or $6\frac{4}{3}$.

$= 1\frac{2}{3}$ Subtract the whole numbers. Then subtract the fractions.

Algebraic Fractions Algebraic fractions can be added and subtracted just like numerical fractions.

Example 3 **Find $\frac{5b}{12} + \frac{3b}{12}$. Write in simplest form.**

$\frac{5b}{12} + \frac{3b}{12} = \frac{5b + 3b}{12}$ The denominators are the same. Add the numerators.

$= \frac{8b}{12}$ or $\frac{2b}{3}$ Simplify.

Exercises

Find each sum or difference. Write in simplest form.

1. $\frac{19}{20} - \frac{17}{20}$

2. $\frac{23}{25} - \frac{8}{25}$

3. $\frac{5}{9} - \frac{2}{9}$

4. $\frac{3}{7} - \frac{5}{7}$

5. $\frac{4}{12} - \frac{7}{12}$

6. $\frac{14}{15} - \frac{9}{15}$

7. $\frac{4c}{8} + \frac{2c}{8}$

8. $\frac{8x}{21} - \frac{11x}{21}$

9. $\frac{9r}{p} - \frac{5r}{p}, p \neq 0$

10. $\frac{10m}{18} + \frac{5m}{18}$

11. $\frac{3t}{16} - \frac{7t}{16}$

12. $\frac{8g}{15} + \frac{g}{15}$

Evaluate each expression if $a = 6\frac{7}{20}$, $b = 3\frac{11}{20}$, and $c = 5\frac{3}{20}$.

13. $a - b$

14. $b - a$

15. $c - a$

16. $b - c$

3-6 Study Guide and Intervention

Adding and Subtracting Unlike Fractions

Add Unlike Fractions Fractions with different denominators are called **unlike fractions.** To add fractions with unlike denominators, rename the fractions with a common denominator. Then add and simplify.

Example 1 **Find $\frac{4}{7} + \frac{1}{3}$. Write in simplest form.**

$\frac{4}{7} + \frac{1}{3} = \frac{4}{7} \cdot \frac{3}{3} + \frac{1}{3} \cdot \frac{7}{7}$ Use 7 · 3 or 21 as the common denominator.

$= \frac{12}{21} + \frac{7}{21}$ Rename each fraction with the common denominator.

$= \frac{19}{21}$ Add the numerators.

Example 2 **Find $-5\frac{5}{6} + 3\frac{5}{8}$. Write in simplest form.**

$-5\frac{5}{6} + 3\frac{5}{8} = \frac{-35}{6} + \frac{29}{8}$ Write the mixed numbers as improper fractions.

$= \frac{-35}{6} \cdot \frac{4}{4} + \frac{29}{8} \cdot \frac{3}{3}$ The LCD for 6 and 8 is 24.

$= \frac{-140}{24} + \frac{87}{24}$ Rename each fraction using the LCD 24.

$= \frac{-53}{24}$ or $-2\frac{5}{24}$ Simplify.

Exercises

Find each sum. Write in simplest form.

1. $\frac{8}{9} + \frac{2}{5}$

2. $-\frac{2}{3} + \frac{1}{4}$

3. $\frac{7}{8} + \frac{1}{4}$

4. $\frac{1}{6} + \left(-\frac{3}{4}\right)$

5. $-\frac{7}{12} + \left(-\frac{3}{5}\right)$

6. $-\frac{1}{3} + \frac{5}{7}$

7. $6\frac{7}{10} + \left(-\frac{2}{3}\right)$

8. $-2\frac{1}{8} + \left(-\frac{3}{4}\right)$

9. $-6\frac{2}{7} + \frac{2}{5}$

10. $3\frac{1}{5} + 2\frac{3}{4}$

11. $7\frac{5}{6} + \left(-3\frac{1}{3}\right)$

12. $6\frac{3}{4} + 3\frac{1}{2}$

13. $7\frac{4}{9} + 9\frac{1}{6}$

14. $-7\frac{1}{2} + \left(-3\frac{2}{9}\right)$

15. $-10\frac{1}{7} + 6\frac{1}{4}$

3-6 Study Guide and Intervention *(continued)*

Adding and Subtracting Unlike Fractions

Subtract Unlike Fractions To subtract fractions with unlike denominators, rename the fractions with a common denominator. Then subtract and simplify.

Example 1 **Find $\frac{4}{9} - \frac{2}{3}$. Write in simplest form.**

$\frac{4}{9} - \frac{2}{3} = \frac{4}{9} - \frac{2}{3} \cdot \frac{3}{3}$ The LCD is 9.

$= \frac{4}{9} - \frac{6}{9}$ Rename using LCD.

$= -\frac{2}{9}$ Simplify.

Example 2 **Find $9\frac{2}{9} - 8\frac{5}{6}$. Write in simplest form.**

$9\frac{2}{9} - 8\frac{5}{6} = \frac{83}{9} - \frac{53}{6}$ Write the mixed numbers as improper fractions.

$= \frac{83}{9} \cdot \frac{2}{2} - \frac{53}{6} \cdot \frac{3}{3}$ Rename fractions using the LCD, 18.

$= \frac{166}{18} - \frac{159}{18}$ Simplify.

$= \frac{7}{18}$ Subtract the numerators.

Exercises

Find each difference. Write in simplest form.

1. $\frac{7}{15} - \frac{3}{10}$

2. $-\frac{6}{11} - \frac{6}{11}$

3. $\frac{13}{15} - \frac{2}{5}$

4. $\frac{3}{8} - \frac{1}{12}$

5. $-\frac{7}{9} - \frac{4}{5}$

6. $\frac{5}{12} - \left(-\frac{3}{8}\right)$

7. $\frac{5}{6} - \frac{7}{10}$

8. $-\frac{2}{5} - \frac{6}{8}$

9. $\frac{7}{10} - \frac{3}{4}$

10. $4\frac{3}{10} - \left(-2\frac{4}{5}\right)$

11. $4\frac{1}{6} - 3\frac{1}{8}$

12. $5\frac{8}{9} - \left(-2\frac{1}{3}\right)$

13. $5\frac{1}{10} - 3\frac{2}{3}$

14. $-6\frac{3}{5} - \left(-2\frac{1}{4}\right)$

15. $10\frac{5}{6} - \left(-5\frac{2}{3}\right)$

4-1 Study Guide and Intervention

The Distributive Property

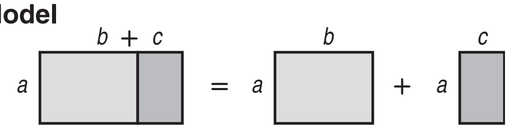
Numerical Expressions The expressions $2(1 + 5)$ and $2 \cdot 1 + 2 \cdot 5$ are equivalent expressions because they have the same value, 12. The **Distributive Property** combines addition and multiplication.

Symbols

$a(b + c) = ab + ac$

$(b + c)a = ab + ac$

Model



The Distributive Property also combines subtraction and multiplication.

Symbols

$a(b - c) = ab - ac$

$(b - c)a = ab - ac$

Example 1 **Use the Distributive Property to write 2(6 + 3) as an equivalent expression. Then evaluate the expression.**

$2(6 + 3) = 2 \cdot 6 + 2 \cdot 3$

$= 12 + 6$ Multiply.

$= 18$ Add.

Example 2 **Use the Distributive Property to write 5(9 − 3) as an equivalent expression. Then evaluate the expression.**

$5(9 - 3) = 5 \cdot 9 - 5 \cdot 3$

$= 45 - 15$ Multiply.

$= 30$ Subtract.

Exercises

Use the Distributive Property to write each expression as an equivalent expression. Then evaluate the expression.

1. $3(8 + 2)$

2. $2(9 + 11)$

3. $5(19 - 6)$

4. $-6(3 + 14)$

5. $(17 - 4)3$

6. $(5 + 3)7$

7. $9(20 + 8)$

8. $(8 - 3)4$

9. $7(40 - 5)$

4-1 Study Guide and Intervention *(continued)*

The Distributive Property

Algebraic Expressions The Distributive Property can also be used with algebraic expressions containing variables.

Example 1 **Use the Distributive Property to write $7(m + 5)$ as an equivalent algebraic expression.**

$7(m + 5) = 7m + 7 \cdot 5$

$= 7m + 35$ Simplify.

Example 2 **Use the Distributive Property to write $3(n - 8)$ as an equivalent algebraic expression.**

$3(n - 8) = 3[n + (-8)]$ Rewrite $n - 8$ as $n + (-8)$.

$= 3n + 3 \cdot (-8)$ Distributive Property

$= 3n + (-24)$ Simplify.

$= 3n - 24$ Definition of subtraction

Exercises

Use the Distributive Property to write each expression as an equivalent expression.

1. $3(d + 4)$

2. $(w - 5)4$

3. $-2(c + 7)$

4. $9(b + 4)$

5. $(p - 10)8$

6. $-11(g - 6)$

7. $-14(j + 3)$

8. $(15 - a)20$

9. $9(50 + h)$

10. $5(12 - c)$

11. $-12(s - 2)$

12. $8(x + 60)$

13. $(y - 13)20$

14. $-15(4 + n)$

15. $7(r - 11)$

4-2 Study Guide and Intervention

Simplifying Algebraic Expressions

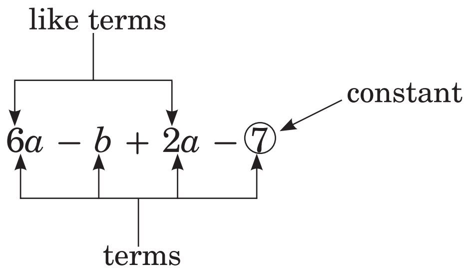
Parts of Algebraic Expressions

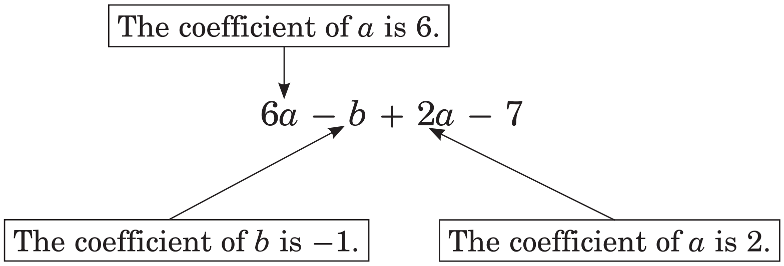
term: a number, variable, or a product of numbers and variables; terms in an expression are seperated by addition or subtraction signs

coefficient: the numerical part of a term that also contains a variable

constant: term without a variable

like terms: terms that contain the same variables





Example **Identify the terms, like terms, coefficients, and constants in the expression $4m - 5m + n - 7$.**

$4m - 5m + n - 7 = 4m + (-5m) + n + (-7)$ Definition of Subtraction

$= 4m + (-5m) + 1n + (-7)$ Identity Property

The terms are $4m$, $-5m$, and $1n$. The like terms are $4m$ and $-5m$. The coefficients are 4, −5, and 1. The constant is −7.

Exercises

Identify the terms, like terms, coefficients, and constants in each expression.

1. $2 + 6a + 4a$

2. $m + 4m + 2m + 5$

3. $3c + 4d - c + 2$

4. $5h - 3g + 2g - h$

5. $3w + 4u - 6$

6. $4r - 5s + 5s - 2r$

7. $-4r - 7 + 6r - s$

8. $-12 - 8x + 8x - 2z$

9. $\frac{4}{7}a + \frac{3}{7}b + \frac{1}{5}a$

4-2 Study Guide and Intervention *(continued)*

Simplifying Algebraic Expressions

Simplify Algebraic Expressions When an algebraic expression has no like terms and no parentheses, we say that it is in **simplest form.**

To make it easier to simplify an algebraic expression, rewrite subtraction as addition. Then use the Commutative Property to group like terms together.

Example 1 **Simplify $6x - 5 - x + 7$.**

$6x - 5 - x + 7 = 6x + (-5) + (-x) + 7$	Definition of Subtraction
$= 6x + (-5) + (-1x) + 7$	Identity Property
$= 6x + (-1x) + (-5) + 7$	Commutative Property
$= 5x + 2$	Simplify.

Example 2 **Simplify $5t - 7(s - 4t)$.**

$5t - 7(s - 4t) = 5t + (-7)[s + (-4t)]$	Definition of Subtraction
$= 5t + (-7s) + (-7 \cdot -4)t$	Distributive Property
$= 5t + (-7s) + 28t$	Simplify.
$= 5t + 28t + (-7s)$	Commutative Property
$= 33t + (-7s)$ or $33t - 7s$	Simplify.

Exercises

Simplify each expression.

1. $9m + 3m$
2. $5x - x$
3. $8y + 2y + 3y$
4. $4 + m - 3m$

5. $13a + 7a + 2a$
6. $3y + 1 + 5 + 4y$
7. $8d - 4 - d + 5$
8. $10 - 4s + 2s - 3$

9. $-15e + 7 - 5e - 9$
10. $-8(r + 6) - r + 1$
11. $-12c + 3 - 9(11 - c)$

12. $4.3x - 8.1 + 0.2x - 17.5$
13. $-7.6 - 9y - 6.5 + 4.7y$
14. $-0.3g - 4.2 + 6.1g - 0.9$

15. $\frac{1}{5}(p - 10) + 13p - 7$
16. $(a + 12)\frac{5}{6} - 5a + 11$
17. $-6h - 5 + \frac{2}{3}(24h - 12)$

18. $7h - 8(2g - 3h)$
19. $-6n + 3(4p + 2n)$
20. $(-2f + e)5 - 12f$

NAME ______________________ DATE __________ PERIOD __________

4-3 Study Guide and Intervention

Solving Equations by Adding or Subtracting

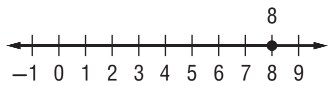
Properties of Equality An **equation** is a mathematical sentence with an equals sign showing that the expressions on either side are equal. **Inverse operations** can be used to find the **solution**, or the value of the variable which makes the equation true. Addition and subtraction are inverse operations.

Addition Property of Equality	Subtraction Property of Equality
If you add the same number to both sides of an equation, the two sides remain equal.	If you subtract the same number from both sides of an equation, the two sides remain equal.

Example **Solve each equation. Check your solution and graph it on a number line.**

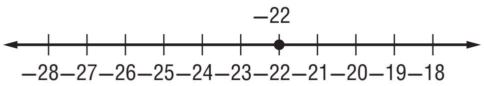
a. $x - 2 = 6$

$x - 2 = 6$	Write the equation.
$+ 2 = + 2$	Addition Property of Equality
$x - 0 = 8$	Additive Inverse Property; $-2 + 2 = 0$
$x = 8$	Identity Property; $x + 0 = x$
CHECK: $x - 2 = 6$	Write the equation.
$8 - 2 \stackrel{?}{=} 6$	Check to see whether this sentence is true.
$6 = 6$ ✓	The sentence is true.



b. $-13 = x + 9$

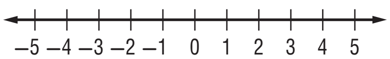
$-13 = x + 9$	Write the equation.
$-9 = \quad -9$	Subtraction Property of Equality
$-22 = x + 0$	Additive Inverse Property; $-9 + 9 = 0$
$-22 = x$	Identity Property; $x + 0 = x$
CHECK: $-13 = x + 9$	Write the equation.
$-13 \stackrel{?}{=} -22 + 9$	Check to see whether this sentence is true.
$-13 = -13$ ✓	The sentence is true.



Exercises

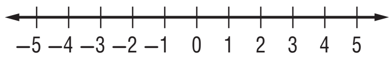
Solve each equation. Check your solution and graph it on a number line.

1. $x + 5 = 2$

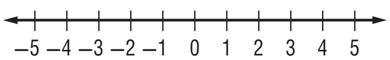


2. $11 + w = 10$

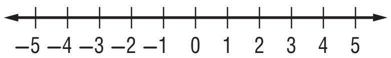
3. $k + 3 = -1$



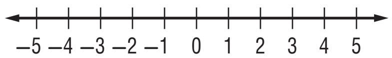
4. $m - 2 = 3$



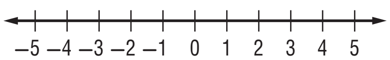
5. $a - 7 = -5$



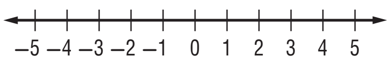
6. $b - 13 = -13$



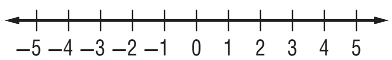
7. $-3 + h = -7$



8. $-12 = y - 9$



9. $2 + r = -3$



4-3 Study Guide and Intervention *(continued)*

Solving Equations by Adding or Subtracting

Write Addition and Subtraction Equations You can write equations to represent word problems. Choose a variable to represent the value you need to find.

Example **SAVINGS Jordan deposited $27.50 into his bank account. Now he has a total of $98.50 in his account. Write and solve an addition equation to find how much Jordan had in his account before he made the deposit.**

Words	amount deposited + amount in bank before the deposit = total after deposit				
Variable	Let a = amount in bank before the deposit.				
Equation	27.50	+	a	=	98.50

Equation $27.50 + a = 98.50$

$27.50 + a = 98.50$ Write the equation.

$27.50 - 27.50 + a = 98.50 - 27.50$ Subtraction Property of Equality

$a = \$71.00$

CHECK: $27.50 + a = 98.50$ Write the equation.

$27.50 + 71.00 \stackrel{?}{=} 98.50$ Check to see whether this sentence is true.

$98.50 = 98.50$ ✓ The sentence is true.

Exercises

1. **MUFFINS** Bonita used some flour to make muffins. The flour bag is now $\frac{1}{3}$ full. The flour bag was $\frac{5}{6}$ full before Bonita made the muffins. Write and solve an addition equation to find what fraction of the flour Bonita used for the muffins.

2. **TEMPERATURE** The high temperature on Wednesday was 56.8°F. The next day, the high temperature was 41.9°F. Write and solve a subtraction equation to find the difference between the two high temperatures.

3. **DVD** The sales price for a DVD player was $89. After tax, Jenna paid a total of $95.46. Write and solve an addition equation to find the amount of the tax.

4. **TESTS** On the first math test of the quarter, Lenny scored 11 points less than he did on the second math test of the quarter. Lenny scored 98 points on the second math test. Write and solve a subtraction equation to find Lenny's score on the first test.

5. **JOGGING** Lanie jogs $1\frac{1}{4}$ miles each morning. She jogs again each afternoon. Lanie jogs a total of $2\frac{7}{10}$ miles every day. Write and solve an addition equation to find how many miles Lanie jogs every afternoon.

NAME ______________________ DATE ____________ PERIOD ____________

4-4 Study Guide and Intervention

Solving Equations by Multiplying or Dividing

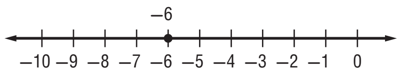
Solve Equations by Dividing Just as addition and subtraction are inverse operations, multiplication and division are inverse operations. To isolate a variable in an equation involving multiplication, you can apply the Division Property of Equality.

> **Division Property of Equality**
>
> If you divide each side of an equation by the same nonzero number, the two sides remain equal.

Example **Solve $-7x = 42$. Check your solution and graph it on a number line.**

$-7x = 42$	Write the equation.
$\frac{-7x}{-7} = \frac{42}{-7}$	Division Property of Equality
$1x = -6$	$-7 \div -7 = 1$, $42 \div -7 = -6$
$x = -6$	Identity Property; $1x = x$
CHECK: $-7x = 42$	Write the equation.
$-7(-6) \stackrel{?}{=} 42$	Replace x with -6 and check to see if the sentence is true.
$42 = 42$ ✓	The sentence is true.

The solution is –6.



To graph -6, draw a dot at -6 on the number line.

Exercises

Solve each equation. Check your solution.

1. $-3a = 15$

2. $-t = 5$

3. $7r = 28$

4. $24 = -8m$

5. $-11b = 44$

6. $12d = -48$

7. $-10p = 10$

8. $-11w = -33$

9. $12g = 42$

10. $-11r = 121$

11. $6d = 126$

12. $12b = 108$

13. $0.4m = 20.4$

14. $-0.7y = 8.4$

15. $0.9t = 0.63$

4-4 Study Guide and Intervention *(continued)*

Solving Equations by Multiplying or Dividing

Solve Equations by Multiplying To isolate a variable in an equation in which a variable is divided, you can apply the Multiplication Property of Equality.

Multiplication Property of Equality
If you multiply each side of an equation by the same number, the two sides remain equal.

Example 1 **Solve $\frac{y}{2} = -2$. Check your solution.**

$\frac{y}{2} = -2$	Write the equation.
$2 \cdot \frac{y}{2} = 2 \cdot (-2)$	Multiplication Property of Equality
$1y = -4$	Multiplicative Inverse Property; $2 \cdot \frac{1}{2} = 1$
$y = -4$	Identity Property. Check your solution.

Example 2 **Solve $-\frac{5}{6}b = 15$. Check your solution.**

$-\frac{5}{6}b = 15$	Write the equation.
$-\frac{6}{5}\left(-\frac{5}{6}\right)b = -\frac{6}{5}\left(\frac{15}{1}\right)$	Multiply each side by $-\frac{6}{5}$, which is the reciprocal of $-\frac{5}{6}$.
$1b = -18$	Multiplicative Inverse Property; $-\frac{6}{5}\left(-\frac{5}{6}\right) = 1$
$b = -18$	Identity Property. Check your solution.

Exercises

Solve each equation. Check your solution.

1. $-1 = \frac{n}{4}$

2. $0 = \frac{h}{7}$

3. $\frac{a}{-2} = -1$

4. $\frac{r}{-5} = -1$

5. $\frac{a}{5} = 22$

6. $\frac{1}{4}q = 8$

7. $\frac{-t}{10} = -14$

8. $\frac{-m}{6} = -12$

9. $\frac{3}{8}j = 18$

10. $\frac{-2}{3}g = 30$

11. $\frac{7}{8}k = 49$

12. $\frac{v}{-15} = 4$

13. $\frac{9}{11}p = 72$

14. $\frac{-w}{25} = 25$

15. $\frac{4}{5}f = 64$

4-5 Study Guide and Intervention

Solving Two-Step Equations

Solve Two-Step Equations A two-step equation contains two operations. To solve two-step equations, use inverse operations to undo each operation in reverse order. First, undo addition/subtraction. Then, undo multiplication/division.

Example 1 **Solve $\frac{c}{2} - 13 = 7$. Check your solution.**

$\frac{c}{2} - 13 = 7$ Write the equation.

$\frac{c}{2} - 13 + 13 = 7 + 13$ Addition Property of Equality

$\frac{c}{2} = 20$ Simplify.

$2 \cdot \frac{c}{2} = 2 \cdot 20$ Multiplication Property of Equality

$c = 40$

CHECK: $\frac{c}{2} - 13 = 7$

$\frac{40}{2} - 13 \stackrel{?}{=} 7$

$20 - 13 \stackrel{?}{=} 7$

$7 = 7$ ✓

Example 2 **Solve $7y - 2y + 4 = 29$. Check your solution.**

$7y - 2y + 4 = 29$ Write the equation.

$5y + 4 = 29$ Combine like terms.

$-4 = -4$ Subtraction Property of Equality

$5y = 25$ Simplify.

$\frac{5y}{5} = \frac{25}{5}$ Division Property of Equality

$y = 5$ Simplify. Check your solution.

CHECK: $7y - 2y + 4 = 29$

$7(5) - 2(5) + 4 \stackrel{?}{=} 29$

$35 - 10 + 4 \stackrel{?}{=} 29$

$25 + 4 \stackrel{?}{=} 29$

$29 = 29$ ✓

Exercises

Solve each equation. Check your solution.

1. $5t + 2 = 7$ **2.** $2x + 5 = 9$ **3.** $6u - 8 = 28$ **4.** $8m - 7 = 17$

5. $\frac{m}{7} - 9 = 5$ **6.** $\frac{k}{9} - 3 = -11$ **7.** $13 + \frac{a}{4} = -3$ **8.** $-3 + \frac{c}{2} = 12$

9. $7 - h = 209$ **10.** $-g + 18 = -32$ **11.** $15 - p = 3$ **12.** $-\frac{2}{5}c - 8 = 32$

13. $\frac{3}{8}q + 12 = 36$ **14.** $3 - \frac{3}{4}n = 9$ **15.** $\frac{7}{9}v + 2 = 23$ **16.** $7 + \frac{1}{8}l = -2$

17. $\frac{v}{-3} + 8 = 22$ **18.** $8x - 16 + 8x = 16$ **19.** $12a - 14a = 8$ **20.** $7c - 8 - 2c = 17$

21. $6 = -y + 42 - 2y$ **22.** $16 + 8r - 4r + 4 = 24$

4-5 Study Guide and Intervention *(continued)*

Solving Two-Step Equations

Solve Real-World Problems When solving two-step equations, always remember to add or subtract first and then multiply or divide to isolate the variable. This is the opposite of the order of operations.

Example **Nina read 50 pages of a 485-page book. Nina now plans to read 15 pages a day. The equation $50 + 15x = 485$ represents how many days it will take Nina to read the rest of the book. Write the steps that can be used to solve the equation.**

$50 + 15x = 485$	Write the equation.
$50 + 15x = 485$	
$\underline{-50 \qquad = -50}$	Subtraction Property of Equality
$15x = 435$	Simplify.
$\frac{15x}{15} = \frac{435}{15}$	Division Property of Equality
$x = 29$	Simplify.

To solve the equation, first subtract 50 and then divide by 15.

CHECK: $50 + 15x = 485$	Write the equation.
$50 + 15(29) \stackrel{?}{=} 485$	Substitute the solution for *x*.
$50 + 435 \stackrel{?}{=} 485$	Multiply.
$485 = 485$ ✓	Add.

Exercises

1. **FUNDRAISING** A high school band needs $1,200 for a trip. So far they have raised $430. They have 5 more fundraisers planned. The equation $\$430 + 5f = \$1,200$ represents how much money they must raise at each of the remaining fundraisers. List the series of steps you would take to solve the equation. Then give the solution.

2. **PRINTS** Haley bought a membership to an online photo-sharing site for $12. After purchasing the membership, she wanted to buy several prints. Prints cost $0.12 each. She has a total of $18.00 to spend on both the membership and the prints. The equation $\$12 + \$0.12p = \$18$ represents how many prints Haley can purchase. List the series of steps you would take to solve the equation. Then give the solution.

3. **SAVINGS** Tim has $85. He wants to save more money to buy a game system for $390. He is able to save $20 a week. The equation $\$85 + 20w = \$390$ represents how many weeks Tim must save. List the series of steps you would take to solve the equation. Then give the solution.

4. **CELL PHONES** A cell phone plan costs $14.75 per month, plus $0.18 cents per minute. Lisa has budgeted $35 a month for her cell phone. The equation $\$14.75 + 0.18m = \$35$ represents how many minutes Lisa can use each month. List the series of steps you would take to solve the equation. Then give the solution.

4-6 Study Guide and Intervention

Writing Equations

Write Two-Step Equations Just as phrases can be represented as expressions, sentences can be represented as equations.

Phrase: Two more than three times a number.
Expression: $2 + 3n$
Sentence: Two more than three times a number is 11.
Equation: $2 + 3n = 11$

Example **Clint has 95 trading cards. This is 17 more than three times the number of cards his brother Wyatt has.**

Words	Three times Wyatt's cards + 17 = Clint's cards		
Symbols	Let w = Wyatt's cards.		
Equation	$3w$	$+ 17 =$	95

Exercises

Translate each sentence into an equation.

1. Nine more than half of a number is 21.
2. Six fewer than $\frac{1}{3}$ of a number is 27.
3. Eleven more than three times a number is 101.
4. The quotient of a number and four decreased by 2 is 6.
5. Julie has 66 stuffed animals which is 8 fewer than twice the number of stuffed animals that Carly has.
6. The $22 Mara spent at a museum gift shop was $4 more than twice the admission to the museum.
7. A hamburger costs $7 which is $2 more than one-third the cost of a pizza.
8. Riley lives 62 miles from his grandma's house which is 22 miles farther than one-quarter the distance to his aunt's house.
9. Angie is 11, which is 3 years younger than 4 times her sister's age.
10. A puppy weighs 14 pounds which is 6 more than one-fifth the mother dog's weight.

4-6 Study Guide and Intervention *(continued)*

Writing Equations

Two-Step Verbal Problems Some real-world situations involve a given amount which then increases or decreases at a certain rate. Such a situation can be represented by a two-step equation.

Example **PRINTING A laser printer prints 9 pages per minute. Liza refilled the paper tray after it had printed 92 pages. In how many more minutes will there be a total of 245 pages printed?**

Understand You know the number of pages printed and the total number of pages to be printed. You need to find the number of minutes required to print the remaining pages.

Plan Let m = the number of minutes. Write and solve an equation. The remaining pages to print is $9m$.
remaining pages + pages printed = total pages
$$9m \quad + \quad 92 \quad = \quad 245$$

Solve

$9m + 92 = 245$	Write the equation.
$9m + 92 - 92 = 245 - 92$	Subtraction Property of Equality
$9m = 153$	Simplify.
$m = 17$	Division Property of Equality

Check The remaining 153 pages will print in 17 minutes. Since 245 – 153 = 92, the answer is correct.

Exercises

Solve each problem by writing and solving an equation.

1. **METEOROLOGY** During one day in 1918, the temperature in Granville, North Dakota, began at −33° and rose for 12 hours. The high temperature was about 51°. About how many degrees per hour did the temperature rise?

2. **SAVINGS** John has $825 in his savings account. He has decided to deposit $65 per month until he has a total of $1800. In how many months will this occur?

3. **SKYDIVING** A skydiver jumps from an airplane at an altitude of 12,000 feet. After 42 seconds, she reaches 4608 feet and opens her parachute. What was her average velocity during her descent?

4. **FLOODING** The water level of a creek has risen 4 inches above its flood stage. If it continues to rise steadily at 2 inches per hour, how long will it take for the creek to be 12 inches above its flood stage?

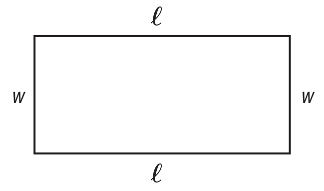
5. **AGES** Maya's brother was 12 when she was born. The sum of their ages is 22. Find their ages.

5-1 Study Guide and Intervention

Perimeter and Area

Perimeter Formulas are equations that show relationships among certain quantities. They usually contain two or more variables. You can use formulas to find the perimeter of a figure. **Perimeter** is the distance around a geometric figure.

Perimeter of a rectangle

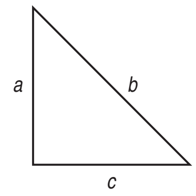


Words The perimeter of a rectangle is the sum of twice the length and twice the width.

Symbols $P = \ell + \ell + w + w$

$P = 2\ell + 2w$ or $2(\ell + w)$

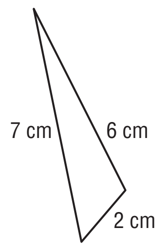
Perimeter of a triangle



Words The perimeter of a triangle is the sum of the measure of all three sides.

Symbols $P = a + b + c$

Example 1 **Find the perimeter of the triangle.**



$P = a + b + c$ Write the formula for perimeter.

$P = 7 + 6 + 2$ Replace a with 7, b with 6, and c with 2.

$P = 15$ cm Simplify. The perimeter is 15 cm.

Example 2 **The perimeter of a rectangle is 26 inches. Its length is 7 inches. Find the width.**

$P = 2\ell + 2w$ Write the formula for perimeter.

$26 = 2 \cdot 7 + 2w$ Replace P with 26, and ℓ with 7.

$26 = 14 + 2w$ Simplify.

$26 - 14 = 14 - 14 + 2w$ Subtraction Property of Equality

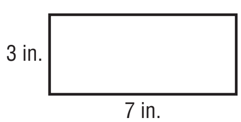
$12 = 2w$ Simplify.

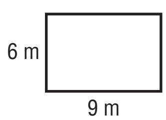
$\frac{12}{2} = \frac{2w}{2}$ Division Property of Equality

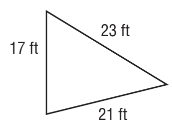
$6 = w$ Simplify. The width of the rectangle is 6 inches.

Exercises

Find the perimeter for each figure.

1. 

2. 

3. 

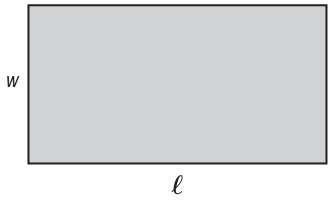
4. Find the length of a rectangle if the width is 4.7 meters and the perimeter is 12.6 meters.

5-1 Study Guide and Intervention *(continued)*

Perimeter and Area

Area Formulas can also be used to calculate the area of a figure. **Area** is a measure of the surface enclosed by a figure and is always given in square units, u^2.

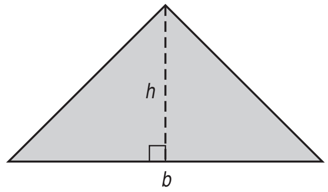
Area of a rectangle



Words The area of a rectangle is the product of the length and width.

Symbols $A = \ell w$

Area of a triangle



Words The area of a triangle is one-half the product of the base and height.

Symbols $A = \frac{1}{2}bh$

Example 1 **The base of a triangle is 14 feet and its height is 4.5 feet. Find its area.**

$A = \frac{1}{2}bh$ Write the formula for area.

$A = \frac{1}{2} \cdot 14 \cdot 4.5$ Replace *b* with 14 and *h* with 4.5.

$A = 31.5$ Simplify. The area is 31.5 square feet.

Example 2 **Find the length of a rectangle with an area of 54 square yards and a width of 8 yards.**

$A = \ell w$ Write the formula for area.

$54 = 8\ell$ Replace *A* with 54 and *w* with 8.

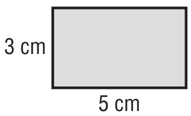
$\frac{54}{8} = \frac{8\ell}{8}$ Division Property of Equality

$6.75 = \ell$ Simplify. The length is 6.75 yards.

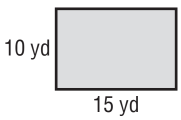
Exercises

Find the area for each figure.

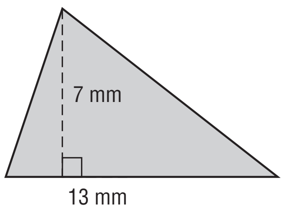
1.



2.



3.



4. Find the height of a triangle if the area is 48 square millimeters and the base is 24 millimeters.

5-2 Study Guide and Intervention

Solving Equations with Variables on Each Side

To solve equations with variables on each side, use the Addition or Subtraction Property of Equality to write an equivalent equation with the variable on one side. Then solve the equation.

Example **Solve the equation $12x - 3 = 4x + 13$. Then check your solution.**

$12x - 3 = 4x + 13$	Write the equation.
$12x - 4x - 3 = 4x - 4x + 13$	Subtract $4x$ from each side.
$8x - 3 = 13$	Simplify.
$8x - 3 + 3 = 13 + 3$	Add 3 to each side.
$8x = 16$	Simplify.
$x = 2$	Mentally divide each side by 8.

To check your solution, replace x with 2 in the original equation.

CHECK	$12x - 3 = 4x + 13$	Write the equation.
	$12(2) - 3 \stackrel{?}{=} 4(2) + 13$	Replace x with 2.
	$24 - 3 \stackrel{?}{=} 8 + 13$	Simplify.
	$21 = 21$ ✓	The statement is true.

Exercises

Solve each equation. Check your solution.

1. $2x + 1 = x + 11$

2. $a + 2 = 5 + 4a$

3. $7y + 25 = 2y$

4. $n + 11 = 2n$

5. $7 - 4c = 3c - 7$

6. $4 - 3b = 6b - 5$

7. $9d - 9 = 3d - 3$

8. $f - 4 = 6f + 26$

9. $-2s + 3 = 5s + 24$

10. $5a - 3 = 8a + 6$

11. $8n - 12 = -12n + 8$

12. $7y + 8 = -2y - 64$

13. $1 + 3x = 7x - 7$

14. $6a - 3 = 4 + 7a$

15. $3b - 1 = 14 + 2b$

16. $12c + 18 = 4 + 5c$

17. $9y + 3 = 5y - 13$

18. $3n - 2 = 5n + 12$

5-2 Study Guide and Intervention *(continued)*

Solving Equations with Variables on Each Side

Write Equations with Variables On Each Side You can write equations with variables on each side to solve word problems.

Example **SHOPPING Maya bought a pair of boots for $32 and then bought 3 T-shirts. Paul bought a cap for $12 and then bought 5 T-shirts. If all the T-shirts cost the same amount, and Maya and Paul spent the same amount in all, write and solve an equation to find the cost of one T-shirt.**

Words	cost of boots	+	number of T-shirts	×	cost per T-shirt	=	cost of cap	+	number of T-shirts	× cost per T-shirt
Variable	Let t	=	the cost of one T-shirt							
Equation	32	+	$3t$			=	12	+	$5t$	

$32 + 3t = 12 + 5t$ Write the equation.

$32 + 3t - 3t = 12 + 5t - 3t$ Subtraction Property of Equality

$32 = 12 + 2t$ Simplify.

$32 - 12 = 12 - 12 + 2t$ Subtraction Property of Equality

$20 = 2t$ Simplify.

$10 = t$ Mentally divide each side by 2.

The cost for one T-shirt is $10.

Exercises

1. **PHONES** Acme Phone Company charges $21 a month plus $0.05 a minute. Belltone Phones charges $15 a month plus $0.11 a minute. Write and solve an equation to determine how many minutes a month you must use for the costs of using either company to be equal.

2. **PARTIES** Mrs. Lin is planning her daughter's birthday party. At Parties R Us, the fee is $80 plus $10 per child. At the Birthday Palace, the fee is $150 plus $5 per child. Write and solve an equation to determine how many children must be invited for the costs to be equal.

3. **POOLS** A town pool has two individual membership rates. You can pay a $75 membership fee and then $2 each time you use the pool or you can pay a $15 membership fee and $5 each time you use the pool. Write and solve an equation to determine how many times you must visit the pool for the costs to be equal.

4. **TAXI** Speedy Cab has an initial charge of $2.50 plus $3.50 for each additional mile. Friendly Cab has an initial charge of $5.50 plus an additional $2.00 per mile. Write and solve an equation to determine how many miles you must go for the costs to be equal.

5-3 Study Guide and Intervention

Inequalities

Write Inequalities A mathematical sentence that contains any of the symbols listed below is called an **inequality.**

$<$	$>$	$\leq$	$\geq$
• is less than • is fewer than	• is greater than • is more than • exceeds	• is less than or equal to • is no more than • is at most	• is greater than or equal to • is no less than • is at least

Example 1 **Write an inequality for the sentence.**

Fewer than 70 students attended the last dance.

Words	*Fewer than* 70 students attended the last dance.
Symbols	Let s = the number of students.
Inequality	$s < 70$

You can substitute a value for a variable in an inequality and determine whether the value makes the inequality true or false.

Example 2 **For the given value, state whether each inequality is *true* or *false*.**

a. $5y - 6 < 14; y = 5$

$5y - 6 < 14$ Write the inequality.

$5(5) - 6 < 14$ Replace the variable with the given value.

$19 < 14$ Simplify.

This sentence is false.

b. $r - 16 \geq -12; r = 4$

$r - 16 \geq -12$

$4 - 16 \geq -12$

$-12 \geq -12$

Although $-12 > -12$ is false, $-12 = -12$ is true. So, this sentence is true.

Exercises

Write an inequality for each sentence.

1. The maximum diving depth is no more than 45 feet below sea level.

2. Adult male elephants can weigh over 12,000 pounds.

3. The maximum fee for any student is $15.

4. You must be at least 38 inches tall to ride the roller coaster.

For the given value, state whether the inequality is *true* or *false*.

5. $m + 8 \geq 5; m = -3$

6. $4 - p < -2; p = 6$

7. $b + 12 \leq 15; b = -1$

8. $j - 7 < -8; j = 0$

NAME ______________________ DATE __________ PERIOD __________

5-3 Study Guide and Intervention *(continued)*

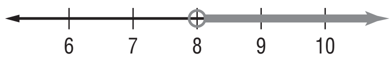
Inequalities

Graph Inequalities Inequalities can be graphed on a number line. This helps you see which values make the inequality true. You can also write inequalities for a graph.

> An *open dot* indicates that the number marked *does not* make the sentence true.
> A *closed dot* indicates that the number marked *does* make the sentence true.
> The direction of the line indicates whether numbers *greater than* or *less than* the number marked make the sentence true.

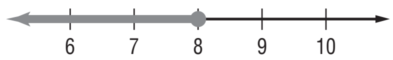
Example 1 **Graph each inequality on a number line.**

a. $x > 8$



The **open dot** means 8 *does not* make the sentence true. The line means that numbers greater than 8 make the sentence true.

b. $x \leq 8$



The **closed dot** means 8 *does* make the sentence true. The line means that numbers less than 8 make the sentence true.

Example 2 **Write an inequality for each graph.**

a.

The open dot means -2 is not included in the graph. The arrow points left, so the graph includes all numbers less than -2.
The inequality is $x < -2$.

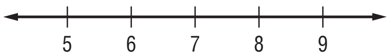
b.

The closed dot means 5 is included in the graph. The arrow points right, so the graph includes all numbers greater than 5.
The inequality is $x \geq 5$.

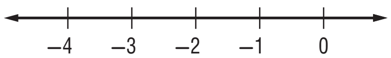
Exercises

Graph each inequality on a number line.

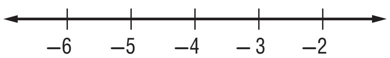
1. $x > 7$



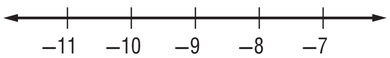
2. $a \leq -2$



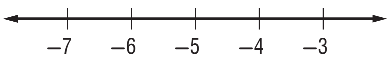
3. $d < -4$



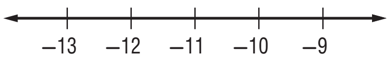
4. $w > -9$



5. $t \geq -5$

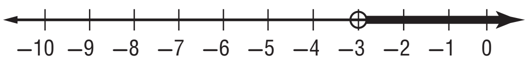


6. $n < -11$



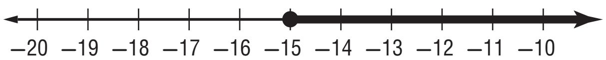
Write the inequality for each graph.

7.



8.

9.



10.

5-4 Study Guide and Intervention

Solving Inequalities

Solve Inequalities by Adding or Subtracting Use the Addition and Subtraction Properties of Inequalities to solve inequalities. When you add or subtract a number from each side of an inequality, the inequality remains true.

Example **Solve $12 + y > 20$. Check your solution.**

$12 + y > 20$	Write the inequality.
$12 - 12 + y > 20 - 12$	Subtraction Property of Inequality
$y > 8$	Simplify.

To check your solution, try any number greater than 8.

CHECK

$12 + y \overset{?}{>} 20$	Write the inequality.
$12 + 9 \overset{?}{>} 20$	Replace y with 9.
$21 > 20$ ✓	This statement is true.

Any number greater than 8 will make the statement true. Therefore, the solution is $y > 8$.

Exercises

Solve each inequality. Check your solution.

1. $-12 < 8 + b$

2. $t - 5 > -4$

3. $p + 5 < -13$

4. $5 > -6 + y$

5. $21 < n - (-18)$

6. $s - 4 \leq 3$

7. $14 > w + (-2)$

8. $j + 6 \geq -4$

9. $z + (-4) < -2.5$

10. $b - \frac{1}{4} < 2\frac{1}{4}$

11. $g - 2\frac{1}{3} \geq 3\frac{1}{6}$

12. $-2 + z < 5$

13. $-10 \leq x - 5$

14. $-23 \geq a + (-6)$

15. $20 < m - 6$

16. $1\frac{1}{2} + b > 7$

17. $k + 5 \geq -7$

18. $-\frac{2}{3} \leq w - 2$

5-4 Study Guide and Intervention *(continued)*

Solving Inequalities

Solve Inequalities by Multiplying or Dividing Use the Multiplication and Division Properties of Inequalities to solve inequalities.

- When you multiply or divide each side of an inequality by a positive number, the inequality remains true. The direction of the inequality sign does not change.
- For an inequality to remain true when multiplying or dividing each side of the inequality by a negative number, however, you must reverse the direction of the inequality symbol.

Example 1 **Solve $8x \geq 72$. Check your solution.**

$8x \geq 72$ Write the inequality.

$\frac{8x}{8} \geq \frac{72}{8}$ Division Property of Inequality

$x \geq 9$ Simplify.

The solution is $x \geq 9$. You can check this solution by substituting 9 or a number greater than 9 into the inequality.

Example 2 **Solve $\frac{y}{-12} < 4$. Then graph the solution on a number line.**

$\frac{y}{-12} < 4$ Write the inequality.

$-12\left(\frac{y}{-12}\right) > -12(4)$ Multiplication Property of Inequality

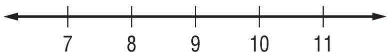
$y > -48$ Simplify.

Graph the solution, $y > -48$.

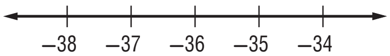
Exercises

Solve each inequality. Then graph the solution on a number line.

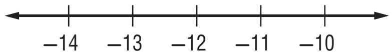
1. $81 < 9d$



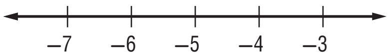
2. $\frac{p}{3} < -12$



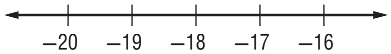
3. $\frac{h}{-4} \geq 3$



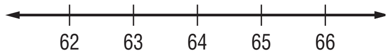
4. $-20t \leq 100$



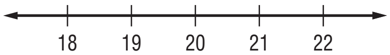
5. $-\frac{2}{3}x > 12$



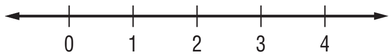
6. $-16 \leq -\frac{1}{4}b$



7. $-8 < \frac{c}{-2.5}$



8. $\frac{n}{3} > 0.5$



5-5 Study Guide and Intervention

Solving Multi-Step Equations and Inequalities

Solve Equations with Grouping Symbols Equations with grouping symbols can be solved by first using the Distributive Property to remove the grouping symbols.

Example 1 **Solve 2(6*m* − 1) = 8*m*. Check your solution.**

$2(6m - 1) = 8m$	Write the equation.
$12m - 2 = 8m$	Use the Distributive Property.
$12m - 12m - 2 = 8m - 12m$	Subtraction Property of Equality
$-2 = -4m$	Simplify.
$\frac{-2}{-4} = \frac{-4m}{-4}$	Division Property of Equality
$\frac{1}{2} = m$	Simplify.

CHECK

$2(6m - 1) = 8m$	
$26\left[\left(\frac{1}{2}\right) - 1\right] \stackrel{?}{=} 8\left(\frac{1}{2}\right)$	Replace *m* with $\frac{1}{2}$.
$2(3 - 1) \stackrel{?}{=} 4$	Simplify.
$4 = 4$ ✓	The solution checks.

No Solution or All Numbers as Solutions Some equations have no solution. The solution set is the **null** or **empty set,** which is represented by Ø. Other equations have every number as a solution. Such an equation is called an **identity.**

Example 2 **Solve each equation.**

a. $2(x - 1) = 4 + 2x$

$2x - 2 = 4 + 2x$

$2x - 2x - 2 = 4 + 2x - 2x$

$-2 = 4$

The solution set is Ø.

b. $-2(x - 1) = 2 - 2x$

$-2x + 2 = 2 - 2x$

$-2x + 2 - 2 = 2 - 2 - 2x$

$-2x = -2x$

$x = x$

The solution set is all real numbers.

Exercises

Solve each equation. Check your solution.

1. $8(g - 3) = 24$ **2.** $5(x + 3) = 25$ **3.** $7(2c - 5) = 7$ **4.** $2(3d + 7) = 5 + 6d$

5. $2(s + 11) = 5(s + 2)$ **6.** $7y - 1 = 2(y + 3) - 2$ **7.** $2(f + 3) - 2 = 8 + 2f$

8. $2(x - 2) + 3 = 2x - 1$ **9.** $1 + 2(b + 6) = 5(b - 1)$ **10.** $2x - 5 = 3(x + 3)$

5-5 Study Guide and Intervention *(continued)*

Solving Multi-Step Equations and Inequalities

Solve Multi-Step Inequalities Some inequalities require more than one step to solve. For such inequalities, undo the operations in reverse order, just as in solving multi-step equations. Remember to reverse the inequality symbol when multiplying or dividing each side of the inequality by a negative number. If the inequality contains parentheses, use the Distributive Property to begin simplifying the inequality.

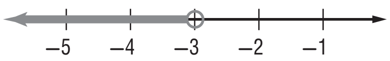
Example **Solve $12 - 2x > 24 + 2x$. Graph the solution on a number line.**

$12 - 2x > 24 + 2x$	Write the inequality.
$12 - 2x - 2x > 24 + 2x - 2x$	Subtraction Property of Inequality
$12 - 4x > 24$	Simplify.
$12 - 12 - 4x > 24 - 12$	Subtraction Property of Inequality
$-4x > 12$	Simplify.
$\frac{-4x}{-4} < \frac{12}{-4}$	Division Property of Inequality
$x < -3$	Simplify.

CHECK

$12 - 2x > 24 + 2x$	Try –4, a number less than –3.
$12 - 2(-4) > 24 + 2(-4)$	Replace *x* with –4.
$12 + 8 > 24 - 8$	Simplify.
$20 > 16$ ✔	The solution checks.

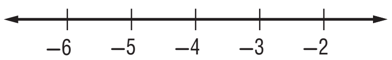
Graph the solution $x < -3$.



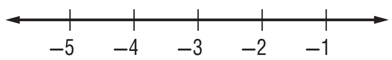
Exercises

Solve each inequality. Graph the solution on a number line.

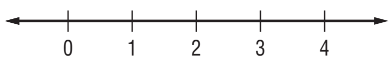
1. $5c + 9 < -11$



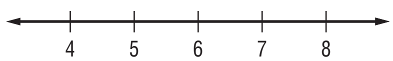
2. $8 - 4p > 20$



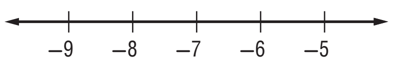
3. $c + 5 \leq 4c - 1$



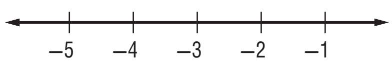
4. $18 - 2n \geq 6$



5. $3(d + 2) < -15$



6. $\frac{b}{3} + 9 > 8$



6-1 Study Guide and Intervention

Ratios

Write Ratios as Fractions in Simplest Form A ratio is a way to compare two quantities using division. Ratios can be written in a number of ways.

The ratio representing 7 out of 12 can be written as: 7 to 12, 7:12, and $\frac{7}{12}$.

Ratios are usually written as fractions in simplest form when the first number being compared is less than the second number being compared.

Example 1 **Express the ratio *16 correct answers out of 20 questions* as a fraction in simplest form. Explain its meaning.**

$$\frac{\text{correct answers}}{\text{number of questions}} = \frac{16}{20} = \frac{4}{5} \quad (\div 4)$$

Divide the numerator and denominator by the GCF, 4.

The ratio of correct answers to questions is 4 to 5. This means that for every 5 questions, 4 were answered correctly. Also, $\frac{4}{5}$ of the questions were answered correctly.

Example 2 **MUSIC Charlize surveyed the sixth graders at her school. Out of 150 students, 55 chose rock as their favorite music. Express this ratio as a fraction in simplest form. Explain its meaning.**

$$\frac{55}{150} = \frac{11}{30} \quad (\div 5)$$

Divide the numerator and denominator by the GCF, 5.

The ratio of sixth graders who chose rock as their favorite music is 11 to 30. This means that for every 30 sixth graders, 11 like rock the best. Also, $\frac{11}{30}$ of sixth graders like rock the best.

Exercises

Express each ratio as a fraction in simplest form.

1. 4 weeks to plan 2 events

2. 9 children to 24 adults

3. 8 teaspoons to 12 forks

4. 16 cups to 10 servings

5. 7 shelves to 84 books

6. 6 teachers to 165 students

7. NEWSPAPER At a newspaper, there are 16 photographers and 84 writers. Express the ratio of photographers to writers as a fraction in simplest form. Explain its meaning.

NAME ______________________ DATE ____________ PERIOD ____________

6-1 Study Guide and Intervention *(continued)*

Ratios

Simplify Ratios Involving Measurements When a ratio involves measurements, both quantities must have the same unit of measure. When the quantities have different units of measure, you must convert one unit to the other. It is usually easiest to convert the larger unit to the smaller unit.

Example **Express the ratio *6 feet to 15 inches* as a fraction in simplest form.**

$\frac{6 \text{ feet}}{15 \text{ inches}}$ Write the ratio as a fraction.

$= \frac{72 \text{ inches}}{15 \text{ inches}}$ Convert 6 feet to 72 inches.

$= \frac{\cancel{72}^{24} \ \cancel{\text{inches}}}{\cancel{15}_{5} \ \cancel{\text{inches}}}$ Divide the numerator and denominator by the GCF, 3.

$= \frac{24}{5}$

Written as a fraction in simplest form, the ratio is 24 to 5.

Exercises

Express each ratio as a fraction in simplest form.

1. 9 ounces to 12 pounds

2. 16 inches to 5 yards

3. 5 quarts to 2 gallons

4. 8 feet to 4 yards

5. 6 feet to 18 inches

6. 7 pints to 14 cups

7. 14 inches to 3 feet

8. 20 inches to 2 yards

9. 9 feet to 12 inches

10. 4 gallons to 2 quarts

11. 3 pints to 2 quarts

12. 22 ounces to 5 pounds

13. 5 feet to 21 inches

14. 12 quarts to 7 pints

NAME ______________________ DATE ____________ PERIOD ____________

6-2 Study Guide and Intervention

Unit Rates

Find Unit Rates A ratio comparing quantities with different units is called a **rate.**

$$\frac{30 \text{ miles}}{2 \text{ hours}}$$

different kinds of units

A **unit rate** is a rate with a denominator of 1. To change a rate to a unit rate, divide the numerator by the denominator.

$$\frac{30 \text{ miles}}{2 \text{ hours}} = \frac{15 \text{ miles}}{1 \text{ hour}} \quad (\div 2)$$

Example **Express the rate *$10 for 8 fish* as a unit rate. Round to the nearest tenth, if necessary.**

$\frac{10 \text{ dollars}}{8 \text{ fish}}$ Write the ratio as a fraction.

$\frac{10 \text{ dollars}}{8 \text{ fish}} = \frac{1.25 \text{ dollars}}{1 \text{ fish}} \quad (\div 8)$ Divide the numerator and denominator by 8 to get a denominator of 1.

The unit rate is $1.25 per fish.

Exercises

Express each rate as a unit rate. Round to the nearest tenth or nearest cent, if necessary.

1. $58 for 5 tickets

2. $4.19 for 4 cans of soup

3. $274.90 for 6 people

4. 565 miles in 12 hours

5. 237 pages in 8 days

6. $102 dollars over 12 hours

7. 180 words in 5 minutes

8. $6.99 for 5 cans

9. $27.99 for 3 T-shirts

10. $19.95 for 5 pounds

11. 145 miles in 6 hours

12. $94.50 for 7 tickets

NAME ______________________ DATE ____________ PERIOD ____________

6-2 Study Guide and Intervention *(continued)*

Unit Rates

Compare Unit Rates Rewriting rates as unit rates makes it easier to compare rates and determine the best rate. Unit rates can also be used to solve problems.

Example 1 **PARTIES The Party Palace charges $225 for 12 children. Party Party charges $195 for 10 children. Which party place has the lowest cost per child?**

First, find the unit rates for the two party places. Then compare them.

Party Palace

$$\frac{225 \text{ dollars}}{12 \text{ children}} = \frac{18.75 \text{ dollars}}{1 \text{ child}} \quad (\div 2)$$

Party Party

$$\frac{195 \text{ dollars}}{10 \text{ children}} = \frac{19.50 \text{ dollars}}{1 \text{ child}} \quad (\div 10)$$

Divide the numerator and the denominator by the denominator.

The unit rate is $18.75 per child.

The unit rate is $19.50 per child.

Since $18.75 < $19.50, the Party Palace has the better rate per child.

Example 2 **READING Kelani read 98 pages in 4 hours. At this rate, how many pages would she read in 9 hours?**

Find the unit rate. Then multiply this unit rate by 9 to find how many hours it would take Kelani to read 9 pages.

$$98 \text{ pages in } 4 \text{ hours} = \frac{98 \text{ pages} \div 4}{4 \text{ hours} \div 4} \text{ or } \frac{24.5 \text{ pages}}{1 \text{ hour}}$$ Find the unit rate.

$$\frac{24.5 \text{ pages}}{1 \text{ hour}} \cdot 9 \text{ hours} = 220.5$$ Divide out the common units.

Kelani would read 220.5 pages in 9 hours.

Exercises

1. **NECKLACES** Shawna strung 5 necklaces in 2 hours. How many necklaces could she string in 7 hours?

2. **GYM** At Funtimes Gym, eight 1-hour classes cost $96. At Fitness Place, twelve 1-hour classes cost $132. Which gym offers the best rate per hour?

3. **SONGS** Jamie downloaded 8 songs in 3 minutes. At this rate, how many songs could he download in 30 minutes?

4. **BIKING** Gina biked 3 miles in 25 minutes. At this rate, how many miles could she bike in 45 minutes?

6-3 Study Guide and Intervention

Converting Rates and Measurements

Dimensional Analysis The process of including units of measurement as factors when you compute is called **dimensional analysis.**

Example 1 **JETS A jet airline traveled at a rate of 540 miles per hour. Convert 540 miles per hour to miles per minute.**

Step 1 You need to convert miles per hour to miles per minute. Choose a conversion factor that converts hours to minutes, with minutes in the denominator.

$$\frac{\text{miles}}{\cancel{\text{hour}}} \cdot \frac{\cancel{\text{hour}}}{\text{minute}} = \frac{\text{miles}}{\text{minute}}$$

So use $\frac{1\text{ h}}{60\text{ min}}$.

Step 2 Multiply.

$\frac{540\text{ mi}}{1\text{ h}} = \frac{540\text{ mi}}{1\text{ h}} \cdot \frac{1\text{ h}}{60\text{ min}}$ Multiply by $\frac{1\text{ h}}{60\text{ min}}$.

$= \frac{540\text{ mi}}{\cancel{1\text{ h}}} \cdot \frac{\cancel{1\text{ h}}}{60\text{ min}}$ Divide out common units.

$= \frac{540\text{ mi}}{60\text{ min}}$ or 9 miles per minute. So, the jet travels 9 miles per minute.

Example 2 **CHEETAH A cheetah can run short distances at a speed of up to 75 miles per hour. How many feet per second is this?**

You need to convert miles per hour to feet per second.

Use 1 mi = 5280 ft and 1 hour = 3600 s.

$\frac{75\text{ mi}}{1\text{ h}} = \frac{75\text{ mi}}{1\text{ h}} \cdot \frac{5280\text{ ft}}{1\text{ mi}} \cdot \frac{1\text{ h}}{3600\text{ s}}$ Multiply by $\frac{5280\text{ ft}}{1\text{ mi}}$ and $\frac{1\text{ h}}{3600\text{ s}}$.

$= \frac{\overset{5}{\cancel{75}}\ \cancel{\text{mi}}}{1\ \cancel{\text{h}}} \cdot \frac{\overset{22}{\cancel{5280}}\text{ ft}}{1\ \cancel{\text{mi}}} \cdot \frac{1\ \cancel{\text{h}}}{\underset{\underset{1}{\cancel{15}}}{\cancel{3600}}\text{ s}}$ Divide the common factors and units.

$= \frac{110\text{ ft}}{1\text{ s}}$ Simplify. So 75 miles per hour is equivalent to 110 feet per second.

Exercises

1. **BICYCLES** Jake was in a bicycle race. His average speed was 22 miles per hour. At this rate, how many feet per hour did Jake travel?

2. **PANDAS** Giant pandas can spend up to 16 hours a day eating bamboo. How many minutes per day is this?

3. **PLUMBING** Karin discovered that her leaky faucet was leaking 1.25 cups of water an hour. At this rate, how many gallons a day were leaking?

4. **TRAINS** A high speed train can travel at 210 kilometers per hour. To the nearest whole meter, how many meters per second is this?

6-3 Study Guide and Intervention *(continued)*

Converting Rates and Measurements

Convert Between Systems Dimensional analysis can also be used to covert between measurement systems.

Example 1 **Convert 2 gallons to liters. Round to the nearest hundredth.**

Use 1 L ≈ 0.264 gal.

$2 \text{ gal} \approx 2 \text{ gal} \cdot \frac{1 \text{ L}}{0.264 \text{ gal}}$ — Multiply by $\frac{1 \text{ L}}{0.264 \text{ gal}}$.

$\approx 2 \text{ gal} \cdot \frac{1 \text{ L}}{0.264 \text{ gal}}$ — Divide out the common units, leaving the desired unit, liter.

$\approx \frac{2 \text{ L}}{0.264}$ or 7.58 L — Simplify.

So, 2 gallons is approximately 7.58 liters.

Example 2 **EAGLES Bald eagles have a diving speed of up to 100 miles per hour. How many meters per second is this?**

To convert miles to meters, use 1 mi ≈ 1.609 km and 1 km = 1000 m.

To convert hours to seconds, use 1 h = 60 min and 1 min = 60 s.

$$\frac{100 \text{ mi}}{1 \text{ h}} \cdot \frac{1.609 \text{ km}}{1 \text{ mi}} \cdot \frac{1000 \text{ m}}{1 \text{ km}} \cdot \frac{1 \text{ h}}{60 \text{ min}} \cdot \frac{1 \text{ min}}{60 \text{ s}}$$

$$= \frac{100 \cancel{\text{mi}}}{1 \cancel{\text{h}}} \cdot \frac{1.609 \cancel{\text{km}}}{1 \cancel{\text{mi}}} \cdot \frac{1000 \text{ m}}{1 \cancel{\text{km}}} \cdot \frac{1 \cancel{\text{h}}}{60 \cancel{\text{min}}} \cdot \frac{1 \cancel{\text{min}}}{60 \text{ s}}$$ — Divide out the common units.

$$= \frac{160{,}900 \text{ m}}{3600 \text{ s}} \text{ or } \frac{44.69 \text{ m}}{1 \text{ s}}$$ — Multiply, then divide.

Bald eagles have a diving speed of up to 44.69 meters per second.

Exercises

Complete each conversion. Round to the nearest hundredth.

1. 14 m ≈ ■ ft **2.** 30 cm ≈ ■ in. **3.** 300 mi ≈ ■ km **4.** 42 yd ≈■ m

5. 8 L ≈ ■ qt **6.** 6 pt ≈ ■ mL **7.** 22 kg ≈ ■ lb **8.** 3 m ≈ ■ in.

9. SPACE STATION The Russian space station Mir orbited around Earth at a rate of 463 kilometers per minute. To the nearest whole mile, how many miles per hour was this?

10. JETS The world's fastest jet is the Blackbird. It is estimated to reach speeds of over 2200 miles per hour. To the nearest whole meter, how many meters per minute is this?

11. WATER The average American uses about 90 gallons of water per day. How many liters per year is this?

6-4 Study Guide and Intervention

Proportional and Nonproportional Relationships

Identify Proportions Two quantities are **proportional** if they have a constant ratio or rate. If they do not have the same ratio or rate, they are said to be **nonproportional**.

Example 1 **Determine whether the distance traveled is proportional to the time. Explain your reasoning.**

Time (minutes)	1	2	3	4
Distance (yards)	300	600	900	1200

Write the rate of time to distance for each minute in simplest form.

$\frac{1}{300} = \frac{1}{300}$ $\frac{2}{600} = \frac{1}{300}$ $\frac{3}{900} = \frac{1}{300}$ $\frac{4}{1200} = \frac{1}{300}$

Since all rates are equal, the time is proportional to the distance.

Example 2 **Determine whether the number of jumping jacks completed is proportional to the time. Explain your reasoning.**

Jumping Jacks Completed	15	30	40	55	65
Time (seconds)	10	20	30	40	50

Write the ratio of jumping jacks completed to time in simplest form.

$\frac{15}{10} = \frac{3}{2}$ $\frac{30}{20} = \frac{3}{2}$ $\frac{40}{30} = \frac{4}{3}$ $\frac{55}{40} = \frac{11}{8}$ $\frac{65}{50} = \frac{13}{12}$

The rates are not equal. So, the number of jumping jacks is *not* proportional to the time.

Exercises

Determine whether the set of numbers in each table is proportional. Explain.

1.

Cookies	6	9	12	15
Cupcakes	4	6	8	10

2.

Population (100,000)	1.3	2.1	3.3	5.2
Years	1	2	3	4

3.

Miles	43	88	129	145
Hours	1	2	3	4

4.

Trading Cards	16	32	48	64
Packs	2	4	6	8

5.

Questions Answered	8	16	30	42
Minutes	2	8	15	20

6.

Cups of Juice	5	15	25	45
Gallons of Punch	2	6	10	18

7.

Money	180	225	270	360
Hours	20	25	30	40

8.

Pounds	10	25	40	100
Cost	40	95	150	365

9.

Months	1	2	3	4
Days	31	59	90	120

10.

Songs	3	5	7	10
Minutes	9	15	21	30

6-4 Study Guide and Intervention *(continued)*

Proportional and Nonproportional Relationships

Describe Proportional Relationships Proportional relationships can also be described using equations of the form $y = kx$, where k is the constant ratio. The constant ratio is called the **constant of proportionality.**

Example **GEOMETRY The perimeter of a square with a side of 3 inches is 12 inches. A square's perimeter is proportional to the length of one of its sides. Write an equation relating the perimeter of a square to the length of one of its sides. What would be the perimeter of a square with 9-inch sides?**

Find the constant of proportionality between perimeter and side length.

$$\frac{\text{perimeter}}{\text{length of sides}} = \frac{12}{3} \text{ or } 4$$

Words: The perimeter is 4 times the length of a side.
Variable: Let P = perimeter and s = the length of a side.
Equation: $P = 4s$

$P = 4s$ Write the equation.
$P = 4(9)$ Replace s with the length of a side.
$P = 36$ Multiply.

The perimeter of a square with a side of 9 inches is 36 inches.

Exercises

1. **SCHOOL** A school is repainting some of its classrooms. Each classroom is repainted with 5.5 gallons of paint. Write and solve an equation to determine the gallons of paint the school must purchase if they repaint 18 classrooms.

2. **BABYSITTING** Gloria earned \$26 for babysitting 4 hours. Write and solve an equation to determine how much Gloria would earn after babysitting 25 hours.

3. **SHOPPING** Mr. Hager bought 5 pounds of coffee for \$35.75. He wants to buy 22 pounds of coffee for his café. Write and solve an equation to determine how much this will cost.

4. **PAINT** A certain paint color requires 3 quarts of red paint for every 2 gallons. Write and solve an equation to determine how many quarts of red paint are required to mix 9 gallons of the paint.

5. **SEWING** Gwen bought $3\frac{1}{4}$ yards of fabric for \$16.22. Write and solve an equation to determine how much 12 yards would cost.

6. **TRAINS** A train traveled 216 miles in 3 hours. Write and solve an equation to determine how many miles the train could travel in 10 hours.

7. **FERRIS WHEEL** Four hundred thirty-five people can ride a Ferris wheel in 15 minutes. Write and solve an equation to determine how many people can ride the Ferris wheel in 90 minutes.

6-5 Study Guide and Intervention

Solving Proportions

Proportions A proportion is an equation stating that two ratios or rates are equal.

$$\frac{a}{b} = \frac{c}{d}$$

An important property of proportions is that their cross products are equal. You can use this property to solve problems involving proportions.

$$\frac{a}{b} = \frac{c}{d}$$

$$ad = bc$$

Example **Solve the proportion $\frac{14.1}{c} = \frac{3}{4}$.**

$\frac{14.1}{c} = \frac{3}{4}$	
$14.1 \cdot 4 = c \cdot 3$	Cross products.
$56.4 = 3c$	Multiply.
$\frac{56.4}{3} = \frac{3c}{3}$	Divide.
$18.8 = c$	Simplify.

The solution is 18.8.

Exercises

ALGEBRA Solve each proportion.

1. $\frac{x}{9} = \frac{16}{12}$

2. $\frac{32}{28} = \frac{w}{7}$

3. $\frac{5}{u} = \frac{60}{132}$

4. $\frac{36}{21} = \frac{24}{s}$

5. $\frac{a}{64} = \frac{225}{480}$

6. $\frac{42}{w} = \frac{56}{8}$

7. $\frac{1}{10} = \frac{m}{12}$

8. $\frac{5}{3} = \frac{85}{h}$

9. $\frac{24}{g} = \frac{2}{30}$

10. $\frac{f}{21} = \frac{57}{63}$

11. $\frac{22}{z} = \frac{121}{16.5}$

12. $\frac{2}{3} = \frac{k}{12.6}$

13. $\frac{r}{9} = \frac{5}{20}$

14. $\frac{d}{21} = \frac{1.5}{3.5}$

15. $\frac{46}{57.5} = \frac{360}{q}$

16. $\frac{4.2}{4.8} = \frac{d}{80}$

17. $\frac{1}{c} = \frac{4.5}{11.7}$

18. $\frac{0.3}{n} = \frac{4.75}{14.25}$

19. $\frac{9.1}{14.7} = \frac{1.3}{p}$

20. $\frac{0.4}{3} = \frac{y}{98.25}$

21. $\frac{v}{33.44} = \frac{1}{3.2}$

6-5 Study Guide and Intervention *(continued)*

Solving Proportions

Use Proportions to Solve Problems

You can use proportions to solve problems involving two quantities. Just be sure to compare the quantities in the same order.

Example **DRIVING Lori drove 232 miles in 5 hours. At this rate, how long will it take her to drive 580 miles?**

Understand You know how long it took to drive 232 miles. You need to find out how long it will take to drive 580 miles.

Plan Write and solve a proportion using ratios that compare miles to hours. Let h represent the hours it will take to drive 580 miles.

Solve There are two ways to set up the proportion.

One Way		Another Way
$\frac{232}{5} = \frac{580}{h}$		$\frac{232}{580} = \frac{5}{h}$
$232 \cdot h = 5 \cdot 580$	Cross products.	$232 \cdot h = 580 \cdot 5$
$232h = 2900$	Multiply.	$232h = 2900$
$\frac{232h}{232} = \frac{2900}{232}$	Divide.	$\frac{232}{232} = \frac{2900}{232}$
$h = 12.5$	Simplify.	$h = 12.5$

Check Check the cross products. Because $232 \cdot 12.5 = 2900$ and $5 \cdot 580 = 2900$, the answer is correct.

So, it will take 12.5 hours to drive 580 miles at the current rate.

Exercises

1. **FUNDRAISING** A school is running a fundraiser. For every \$75 worth of wrapping paper sold, the school receives \$20. How much wrapping paper must be sold to reach the fund-raising goal of \$2500?

2. **PIZZA** At a pizzeria, a 10-pound bag of shredded cheese can be used to make 32 pizzas. How many pounds would be needed to make 100 pizzas?

3. **MONEY** In 4 weeks, Marlie earned \$550 at her job. Write an equation relating the number of weeks, w, to the number of dollars, d. At this rate, how many weeks would it take Marlie to earn \$5000?

4. **SCIENCE** Mike weighs 90 pounds. On a Web site, he calculated that he would weigh about 15 pounds on the Moon. Write an equation relating pounds on Earth, e, to pounds on the Moon, m. About how many pounds would Mike's dog weigh on the Moon if he weighs 54 pounds on Earth?

NAME ______________________ DATE __________ PERIOD __________

6-6 Study Guide and Intervention

Scale Drawings and Models

Use Scale Drawings and Models **Scale drawings** or **scale models** represent objects that are either too large or too small to be drawn or built in actual size. The measures of objects on a scale drawing or model are proportional to the corresponding measures on the actual object.

The **scale** of a drawing or model is the ratio of a given measure on the drawing or model and the corresponding measure on the actual object. If the measurements are in the same unit, the scale can be written without units. In this case, it is called the **scale factor**.

Example 1 **A map shows a scale of 1 inch = 6 miles. The distance between two places on the map is 4.25 inches. What is the actual distance?**

Let x represent the actual distance. Write and solve a proportion.

map width → / actual width → $\frac{1 \text{ inch}}{6 \text{ miles}} = \frac{4.25 \text{ inches}}{x \text{ miles}}$ ← map width / ← actual width

$1 \cdot x = 6 \cdot 4.25$ Find the cross products.

$x = 25.5$ Simplify.

The actual distance is 25.5 miles.

Example 2 **Sam made a model car that is 9 inches long. The actual car that the model is based on is 13.5 feet long. Find the scale and the scale factor of the model.**

Write the ratio of the model's length to the length of the actual car. Then solve a proportion in which the model's length is 1 inch and the length of the actual car is x feet.

model length → / actual length → $\frac{9 \text{ in.}}{13.5 \text{ ft}} = \frac{1 \text{ in.}}{x \text{ ft}}$ ← model length / ← actual length

$9 \cdot x = 13.5 \cdot 1$ Find the cross products.

$9x = 13.5$ Simplify.

$x = 1.5$ Divide each side by 9. Simplify.

So, the scale is 1 inch = 1.5 feet.

To change this to a scale factor with the same units, first write as a ratio.

scale → 1 inch = 1.5 feet → $\frac{1 \text{ in.}}{1.5 \text{ ft}}$ → $\frac{1 \text{ in.}}{18 \text{ in.}}$ → 1:18 scale factor

Exercises

1. **MAPS** Joanna knows the distance to her grandmother's house is 21 miles. On a map, the distance is 5.25 inches. What is the scale of the map?

2. **HOUSES** Kevin drew a scale drawing of his living room. The actual room is 16 feet long. If the room is 12 inches long in the drawing, what is the scale of the drawing?

3. **DOLLHOUSE** Cindy's dad made her a dollhouse that is a scale model of their house. If their house is 45 feet tall and the model is 15 inches tall, what is the scale of the model?

6-6 Study Guide and Intervention *(continued)*

Scale Drawings and Models

Construct Scale Drawings You can make a scale drawing using a proportion involving the measure on the drawing, the actual measure of the object, and the chosen scale.

Example **CLASSROOMS Ms. Statsky's students are making a scale drawing of their classroom. The actual classroom is 30 feet long and 24 feet wide. Make a scale drawing of the classroom. Use a scale of 0.5 inch = 6 feet. Use $\frac{1}{4}$-inch grid paper.**

Step 1 Find the measure of the room's length on the drawing. Let ℓ represent the length.

drawing length → / actual length → $\frac{0.5 \text{ inch}}{6 \text{ feet}} = \frac{\ell \text{ inches}}{30 \text{ feet}}$ ← drawing length / ← actual length

$0.5 \cdot 30 = 6 \cdot \ell$ Find the cross products.

$15 = 6\ell$ Simplify.

$2.5 = \ell$ Divide each side by 6.

On the drawing, the length is 2.5 inches.

Step 2 Find the measure of the room's width on the drawing. Let w represent the width.

drawing width → / actual width → $\frac{0.5 \text{ inch}}{6 \text{ feet}} = \frac{w \text{ inches}}{24 \text{ feet}}$ ← drawing width / ← actual width

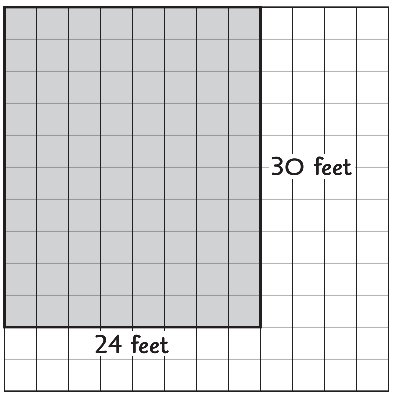
$0.5 \cdot 24 = 6 \cdot w$ Find the cross products.

$12 = 6w$ Simplify.

$2 = w$ Divide each side by 6.

On the drawing, the length is 2 inches.

Step 3 Make the scale drawing.
Use $\frac{1}{4}$-inch grid paper. Since $2\frac{1}{2}$ inches = 10 squares and 2 inches = 8 squares, draw a rectangle that is 10 squares by 8 squares.



Exercises

Make a scale drawing of each of the objects listed below using the given scale. Use $\frac{1}{4}$-inch grid paper.

1. 30-inch by 20-inch table; scale: 0.25 inch = 5 inches

2. 125-foot by 40-foot room; scale: 0.25 inch = 10 feet

3. 6-foot by 12-foot billboard; scale: 0.5 inch = 2 feet

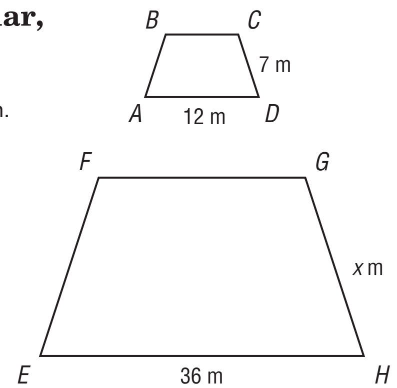
6-7 Study Guide and Intervention

Similar Figures

Corresponding Parts of Similar Figures

Similar figures are figures that have the same shape but not necessarily the same size. If two figures are similar, then the corresponding angles have the same measure, and the corresponding sides are proportional. Because corresponding sides are proportional, you can use proportions or the scale factor to find the measures of the sides of similar figures when some measures are known.

Example **If the polygons *ABCD* and *EFGH* are similar, what is the value of *x*?**

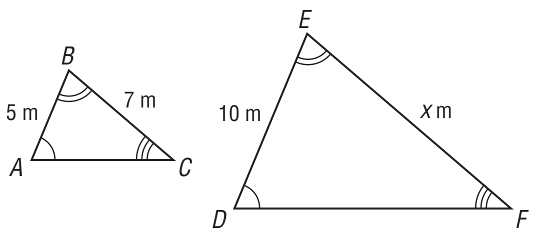


$\frac{AD}{EH} = \frac{CD}{GH}$	The corresponding sides are proportional. Write a proportion.
$\frac{12}{36} = \frac{7}{x}$	Replace *AD* with 12, *EH* with 36, *CD* with 7, and *GH* with *x*.
$12 \cdot x = 36 \cdot 7$	Find the cross products.
$12x = 252$	Simplify.
$x = 21$	Mentally divide each side by 12.

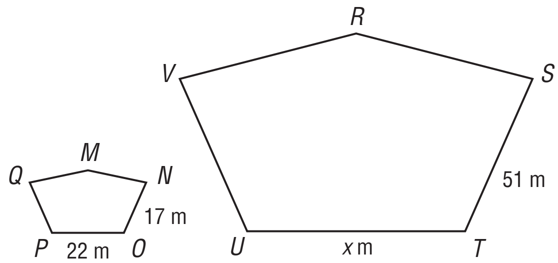
Exercises

The figures are similar. Find each missing measure.

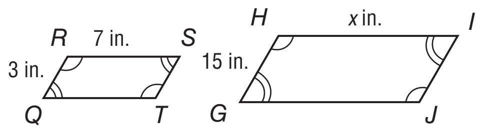
1.



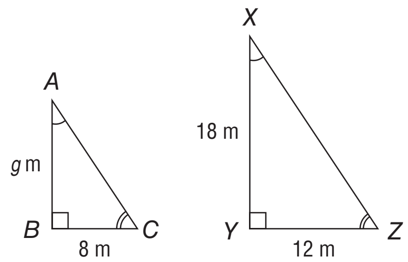
2.



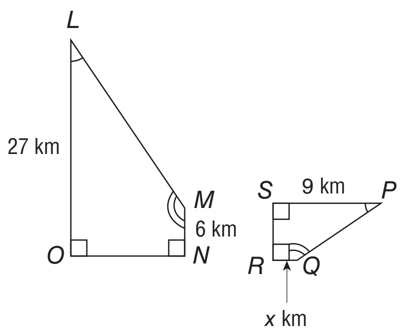
3.



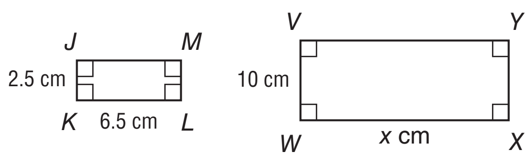
4.



5.



6.

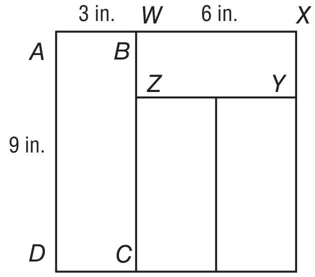


6-7 Study Guide and Intervention *(continued)*

Similar Figures

Scale Factors The scale factor is the ratio of a length on a scale drawing to the corresponding length on the real object. It is also the ratio of corresponding sides in similar figures.

Example **QUILTS Paula is making a quilt. She designed the block shown at the right. If rectangle *ABCD* is similar to rectangle *WXYZ*, find the length of segment *WZ*.**



Find the scale factor from figure *ABCD* to figure *WXYZ* by finding the ratio of corresponding sides with known lengths.

scale factor: $\frac{AD}{WX} = \frac{9}{6}$ or 1.5

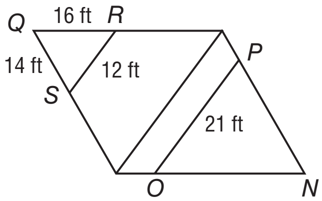
Words	1.5 times a length on figure *WXYZ*	is	a corresponding length on figure *ABCD*
Variable	Let m represent the measure of $\overline{WZ}$.		
Equation	$1.5m = 3$		

$1.5m = 3$ Write the equation.

$m = 2$ Divide each side by 1.5.

So, the length of $\overline{WZ}$ is 2 inches.

Exercises



1. **ART** The art club is painting the mural shown at the right on a wall. Triangle *QRS* and triangle *NOP* are similar.

   a. Find the length of $\overline{NO}$.

   b. Find the length of $\overline{PN}$.

2. **GEOMETRY** Triangle *JKL* is similar to triangle *DEF*. What is the value of $\overline{KL}$ if $\overline{JL}$ is 15 inches, $\overline{DF}$ is 5 inches, and $\overline{EF}$ is 9 inches?

3. **GEOMETRY** Trapezoid *GHIJ* is similar to trapezoid *RSTU*. What is the value of $\overline{ST}$ if $\overline{HI}$ is 6 yards, $\overline{IJ}$ is 9 yards, and $\overline{EF}$ is 27 yards?

4. **GEOMETRY** Rectangle *CDEF* is similar to rectangle *KLMN*. What is the value of $\overline{EF}$ if $\overline{CD}$ is 3 meters, $\overline{KL}$ is 16.5 meters, and $\overline{MN}$ is 38.5 meters?

6-8 Study Guide and Intervention

Dilations

Dilations When you enlarge or reduce a figure by a certain scale factor, the transformation is called a **dilation**. When the center of a dilation on the coordinate plane is the origin, you can find the coordinates of the dilated image by multiplying the coordinates of the original figure by the scale factor. The scale factor is identified as k.

In a dilation with a scale factor of k:

- the dilation is an enlargement if $k > 1$
- the dilation is a reduction if $k < 1$
- to find the new coordinates for vertex (x, y), find (kx, ky)

Example 1 **A figure has vertices $P(1, 1)$, $Q(2, 2)$, $R(5, 3)$, and $S(5, 1)$. Graph the figure and the image of the polygon after a dilation with a scale factor of 2.**

The dilation is $(x, y) \longrightarrow (2x, 2y)$.

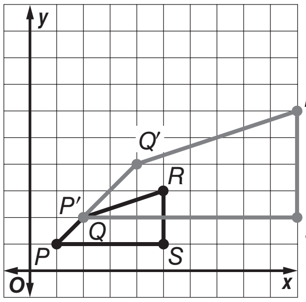
Multiply the coordinates of each vertex by 2. Then graph both figures on the same coordinate plane.

$P(1, 1) \longrightarrow P'(2 \cdot 1, 2 \cdot 1) \longrightarrow P'(2, 2)$

$Q(2, 2) \longrightarrow Q'(2 \cdot 2, 2 \cdot 2) \longrightarrow Q'(4, 4)$

$R(5, 3) \longrightarrow R'(2 \cdot 5, 2 \cdot 3) \longrightarrow R'(10, 6)$

$S(5, 1) \longrightarrow S'(2 \cdot 5, 2 \cdot 1) \longrightarrow S'(10, 2)$



Example 2 **A triangle has vertices $A(15, 12)$, $B(9, 12)$, and $C(9, 6)$. Find the coordinates of the triangle after a dilation with a scale factor of $\frac{1}{3}$.**

The dilation is $(x, y) \longrightarrow (\frac{1}{3}x, \frac{1}{3}y)$. Multiply the coordinates of each vertex by $\frac{1}{3}$.

$A(15, 12) \longrightarrow A'(\frac{1}{3} \cdot 15, \frac{1}{3} \cdot 12) \longrightarrow A'(5, 4)$

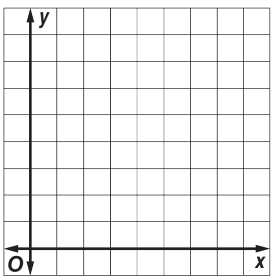
$B(9, 12) \longrightarrow B'(\frac{1}{3} \cdot 9, \frac{1}{3} \cdot 12) \longrightarrow B'(3, 4)$

$C(9, 6) \longrightarrow C'(\frac{1}{3} \cdot 9, \frac{1}{3} \cdot 6) \longrightarrow C'(3, 2)$

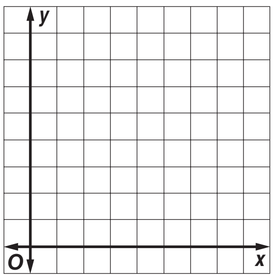
Exercises

Find the vertices of each figure after a dilation with the given scale factor k. Then graph the image.

1. $A(2, 2)$, $B(0, 4)$, $C(4, 8)$, $D(10, 6)$; $k = \frac{1}{2}$



2. $X(1, 0)$, $Y(0, 2)$, $Z(2, 1)$; $k = 3$

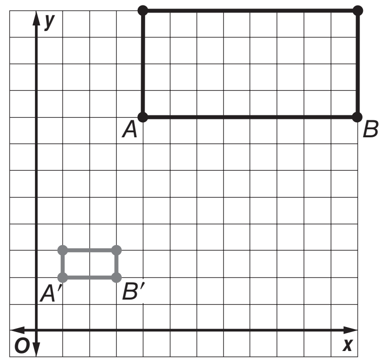


6-8 Study Guide and Intervention *(continued)*

Dilations

Scale Factors When you know the size of a figure and the size of the dilation of that figure, you can determine the scale factor of the dilation.

Example **ART Kiley drew a sketch of the mural that is painted outside her school library. What is the scale factor of the dilation?**



To find the scale factor, write a ratio that compares the length of one side of the original image to the length of the corresponding side of the dilation.

When the image is on a grid, subtract the x-coordinates to find the length.

$$\frac{\text{length on dilation}}{\text{length on original}} = \frac{3 - 1}{12 - 4} \text{ or } \frac{1}{4}$$

So, the scale factor of the dilation is $\frac{1}{4}$.

Exercises

1. **PRESENTS** For his mother's birthday, Paulo wants to enlarge a 3- by 5-inch photo to an 18- by 30-inch photo. What is the scale factor of the dilation?

2. **PHOTOS** Yves found a store that will take a regular photo and transfer an enlarged version of the photo onto a blanket. Yves would like to order one for her grandparents. The photo she chose is 4 by 6 inches. The blanket will be 50 by 75 inches. What is the scale factor of the dilation?

3. **LOGOS** Kevin is using his scanner to make a smaller version of the school logo to put in the yearbook. The original is 7 by 10 inches. The reduced image is 5.25 by 7.5 inches. What is the scale factor of the dilation?

4. **COMPUTERS** Sue is creating a pattern using a computer art program. She made one triangle with a length of 4.2 inches and a height of 6 inches. She duplicated the triangle and reduced it to a length of 2.8 inches and a height of 4 inches. What is the scale factor of the dilation?

5. **KNITTING** Mrs. Gonzalez knit a blanket for her granddaughter Ella. The blanket is 64 by 54 inches. Now Mrs. Gonzales wants to make a blanket for Ella's doll that is 16 by 13.5 inches. What is the scale factor of the dilation?

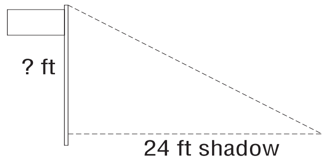
6. **IMAGES** Mr. Chen connected his computer to a projector. His computer screen is 12 inches by 15 inches. The projected image from the screen is 63 inches by 78.75 inches. What is the scale factor of the dilation?

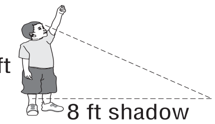
6-9 Study Guide and Intervention

Indirect Measurement

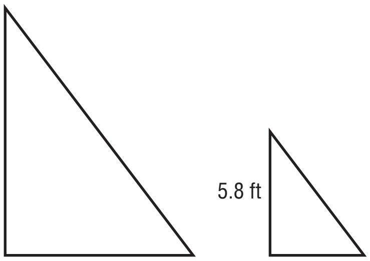
Indirect Measurement The properties of similar triangles can be use to find measurements that are difficult to measure directly. This is called **indirect measurement.**

One type of indirect measurement is *shadow reckoning*. The diagram at the right shows how two objects and their shadows form two sides of similar triangles. You can use a proportion to find measures such as the height of the flag pole.





Example **SCHOOLS A school building in casts a 40.5-foot shadow at the same time a 5.8-foot student casts a 4.4-foot shadow. How tall is the school building to the nearest tenth?**



Understand You know the lengths of the shadows and the height of the student. You need to find the building's height.

Plan To find the height of the building, set up a proportion comparing the student's shadow to the building's shadow. Then solve.

Solve

student's height → 5.8, building's height → h; student's shadow → 4.4, building's shadow → 40.5

$$\frac{5.8}{h} = \frac{4.4}{40.5}$$

$5.8 \cdot 40.5 = h \cdot 4.4$ Find the cross products.

$234.9 = 4.4h$ Multiply.

$53.4 = h$ Divide each side by 4.4.

The height of the school building is 53.4 feet.

Exercises

1. **HOUSES** Lena's house casts a shadow that is 14 feet long at the same time that Lena casts a shadow that is 3.5 feet long. If Lena is 4.5 feet tall, how tall is her house?

2. **ROCKET** Suppose a rocket outside a science museum cast a shadow that was 176 feet. At the same time, a 5.75-foot-tall person standing next to the rocket casts a shadow that is 9.2 feet long. How tall is the rocket?

3. **TOWERS** A cell phone tower casts a shadow that is 92 feet. A building next to the tower is 28 feet high and casts a shadow that is 11.2 feet long. How tall is the cell phone tower?

6-9 Study Guide and Intervention *(continued)*

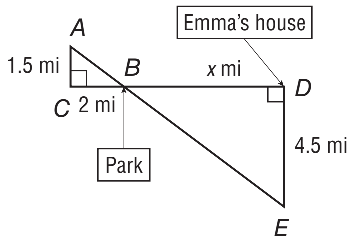
Indirect Measurement

Surveying Methods Another example of indirect measurement involving similar triangles is used by surveyors.

Example **DISTANCES In the figure, $\triangle ABC \sim \triangle EBD$. Find the distance between Emma's house and the park.**

Because the figures are similar, corresponding sides are proportional.

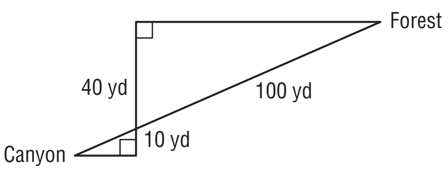
$\frac{CB}{DB} = \frac{AC}{ED}$	Write a proportion.
$\frac{2}{x} = \frac{1.5}{4.5}$	$CB = 2$, $DB = x$, $AC = 1.5$, and $BD = 4.5$
$x \cdot 1.5 = 2 \cdot 4.5$	Cross products.
$1.5x = 9$	Multiply.
$\frac{1.5x}{1.5} = \frac{9}{1.5}$	Divide each side by 1.5.
$x = 6$	Simplify.



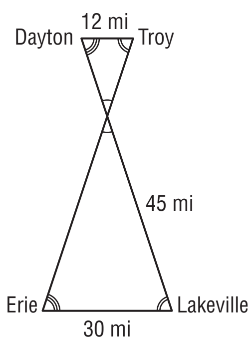
So, the distance between Emma's house and the park is 6 miles.

Exercises

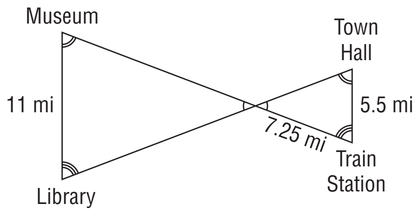
1. **DISTANCES** The triangles below are similar. Find the distance between the canyon and the forest.



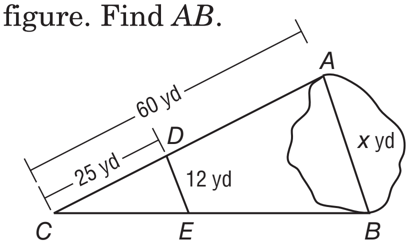
2. **MAPS** The triangles below are similar. Find the distance between Lakeville and Dayton.



3. **DISTANCES** The triangles below are similar. How far is the train station from the museum?



4. **SURVEYING** A surveyor needs to find the distance AB across a pond. He constructs $\triangle CDE$ similar to $\triangle CAB$ and measures the distances as shown on this figure. Find AB.

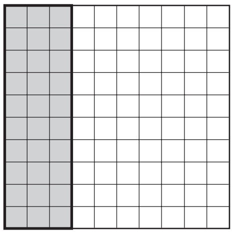


NAME ______________________________ DATE ____________ PERIOD ____________

7-1 Study Guide and Intervention

Fractions and Percents

Percents as Fractions To write a percent as a fraction, express the ratio as a fraction with a denominator of 100. Then simplify if possible.

Percent			
Words	A percent is a part to whole ratio that compares a number to 100.	**Model**	 30 out of 100 = 30%
Examples	30% 30 out of 100 $\frac{30}{100}$		

Example **Write each percent as a fraction in simplest form.**

a. 65%

$65\% = \frac{65}{100}$ Definition of percent

$= \frac{13}{20}$ Simplify.

b. 450%

$450\% = \frac{450}{100}$ Definition of percent

$= \frac{9}{2}$ or $4\frac{1}{2}$ Simplify.

c. $37\frac{1}{2}\%$

$37\frac{1}{2}\% = \frac{37\frac{1}{2}}{100}$ Definition of percent

$= \frac{75}{2} \div 100$ Write $37\frac{1}{2}$ as an improper fraction.

$= \frac{\overset{3}{\cancel{75}}}{2} \cdot \frac{1}{\underset{4}{\cancel{100}}}$ Multiply.

$= \frac{3}{8}$ Simplify.

d. 0.8%

$0.8\% = \frac{0.8}{100}$ Definition of percent

$= \frac{0.8}{100} \cdot \frac{10}{10}$ Multiply by $\frac{10}{10}$ to eliminate the decimal in the numerator.

$= \frac{8}{1000}$ or $\frac{1}{125}$ Simplify.

Exercises

Write each percent as a fraction or mixed number in simplest form.

1. 12% **2.** 5% **3.** 17% **4.** 0.4%

5. 150% **6.** $20\frac{1}{2}\%$ **7.** 98% **8.** 825%

9. 0.6% **10.** 72% **11.** $62\frac{1}{2}\%$ **12.** 1,000%

7-1 Study Guide and Intervention *(continued)*

Fractions and Percents

Fractions as Percents To write a fraction as a percent, write an equivalent fraction with a denominator of 100. If the denominator is not a factor of 100, use a proportion to find what part of 100 the numerator is equal to.

Example 1 **Write each fraction as a percent.**

a. $\frac{7}{10}$

First, find the equivalent fraction with a denominator of 100. Then write the fraction as a percent.

$\frac{7}{10} = \frac{7 \times 10}{10 \times 10} = \frac{70}{100}$ or 70%

So, $\frac{7}{10} = 70\%$.

b. $\frac{9}{4}$

Write an equivalent fraction with a denominator of 100.

$\frac{9}{4} = \frac{9 \times 25}{4 \times 25} = \frac{225}{100} = 225\%$

So, $\frac{9}{4} = 225\%$.

Example 2 **PUPPIES Jonah's dog had 8 puppies. Five of the puppies are female. What percent of the puppies are female?**

To solve, write $\frac{5}{8}$ as a percent.

$\frac{5}{8} = \frac{n}{100}$	Write a proportion using $\frac{n}{100}$.
$5 \cdot 100 = 8 \cdot n$	Cross products
$500 = 8n$	Multiply.
$62\frac{1}{2} = n$	Divide each side by 8.

So, $\frac{5}{8} = 62\frac{1}{2}\%$ or 62.5%.

Exercises

Write each fraction as a percent. Round to the nearest hundredth.

1. $\frac{3}{20}$ **2.** $\frac{2}{5}$ **3.** $\frac{11}{25}$ **4.** $\frac{8}{5}$

5. $\frac{9}{10}$ **6.** $\frac{7}{15}$ **7.** $\frac{5}{12}$ **8.** $\frac{23}{50}$

9. $\frac{9}{16}$ **10.** $\frac{4}{25}$ **11.** $\frac{7}{8}$ **12.** $\frac{9}{40}$

7-2 Study Guide and Intervention

Fractions, Decimals, and Percents

Percents and Decimals When writing a percent as a fraction, the percent is written as a fraction with a denominator of 100. The fraction can be written as a decimal by dividing the numerator of a fraction by its denominator.

$$16\% = \frac{16}{100} = 0.16$$

A decimal can also be written as a fraction and then as a percent.

$$0.09 = \frac{9}{100} = 9\%$$

- To write a percent as a decimal, divide by 100 and remove the percent symbol.
- To write a decimal as a percent, multiply by 100 and add the percent symbol.

Example 1 **Write each percent as a decimal.**

a. 11%

11% = .11 — Remove the % symbol and divide by 100.

= 0.11 — Add a zero in the units place.

b. 0.2%

0.2% = .002 — Remove the % symbol and divide by 100. Add placeholder zeros.

= 0.002 — Add a zero in the units place.

Example 2 **Write each decimal as a percent.**

a. 0.3

0.3 = 0.30 — Multiply by 100. Add a placeholder zero.

= 30% — Add the % symbol.

b. 1.25

1.25 = 1.25 — Multiply by 100.

= 125% — Add the % symbol.

Exercises

Write each percent as a decimal.

1. 12% **2.** 5% **3.** 17% **4.** 72%

5. 150% **6.** 2% **7.** 0.6% **8.** 825%

Write each decimal as a percent.

9. 0.3 **10.** 0.21 **11.** 0.09 **12.** 3.225

13. 0.65 **14.** 0.772 **15.** 0.0015 **16.** 0.01

7-2 Study Guide and Intervention *(continued)*

Fractions, Decimals, and Percents

Compare Fractions, Decimals, and Percents Fractions, decimals, and percents are all different names that represent the same number. You can express a fraction as a percent by first expressing it as a decimal and changing the decimal to a percent. Then you can compare fractions, decimals, and percents by writing them in the same format.

Example 1 **Express each fraction as a percent. Round to the nearest tenth, if necessary.**

a. $\frac{3}{8}$

$\frac{3}{8} = 0.375$ Divide 3 by 8. Multiply 0.375 by 100.

$= 37.5\%$ Add the % symbol.

b. $\frac{5}{9}$

$\frac{5}{9} = 0.555555\ldots$ Divide 5 by 9. Multiply 0.555555… by 100.

$\approx 55.6\%$ Round and add the % symbol.

Example 2 **COMMUNICATION You did a survey in your school and found out that $\frac{19}{50}$ of the students prefer to text message, 29% prefer e-mail, and 0.33 prefer talking on the phone. Which of these groups is the largest?**

Write $\frac{19}{50}$ and 0.33 as percents. Then compare with 29%.

$\frac{19}{50} = 0.38$ or 38% $0.33 = 33\%$

Since 38% is greater than both 33% and 29%, the group that preferred text messaging is the largest.

Exercises

Express each fraction as a percent. Round to the nearest tenth, if necessary.

1. $\frac{33}{40}$ **2.** $\frac{9}{32}$ **3.** $\frac{3}{8}$ **4.** $\frac{11}{4}$

5. $\frac{35}{8}$ **6.** $\frac{1}{5}$ **7.** $\frac{14}{25}$ **8.** $\frac{4}{9}$

9. POLLS In a survey of registered voters, 44% said they would vote for Mr. Johnson, $\frac{2}{5}$ said they would vote for Ms. Smith, and 0.16 said they would vote for Mr. Burns. Which candidate has the largest group of supporters? Explain.

7-3 Study Guide and Intervention

Using the Percent Proportion

Percent Proportion In a **percent proportion**, one ratio compares *part* of a quantity to the *whole* quantity. The other ratio is the equivalent percent, written as a fraction, with a denominator of 100.

Example 1 **Find each percent.**

a. Twelve is what percent of 16?

$\frac{a}{b} = \frac{p}{100} \rightarrow \frac{12}{16} = \frac{p}{100}$ Replace the variables.

$12 \cdot 100 = p \cdot 16$ Find the cross products.

$1200 = 16p$ Multiply.

$75 = p$ Divide.

So, twelve is 75% of 16.

b. What percent of 8 is 7?

$\frac{a}{b} = \frac{p}{100} \rightarrow \frac{7}{8} = \frac{p}{100}$

$p \cdot 8 = 100 \cdot 7$

$700 = 8p$

$87.5 = p$

So, 87.5% of 8 is 7.

Example 2 **Find the part or the whole.**

a. What number is 1.4% of 15?

$\frac{a}{b} = \frac{p}{100} \rightarrow \frac{a}{15} = \frac{p}{100}$ Replace the variables.

$a \cdot 100 = 15 \cdot 14$ Find the cross products

$100a = 21$ Multiply.

$a = 0.21$ Divide.

So, 0.21 is 1.4% of 15.

b. 225 is 36% of what number?

$\frac{a}{b} = \frac{p}{100} \rightarrow \frac{225}{b} = \frac{36}{100}$

$225 \cdot 36 = 100 \cdot b$

$22{,}500 = 36b$

$625 = b$

So, 225 is 36% of 625.

Exercises

Use the percent proportion to solve each problem. Round to the nearest tenth, if necessary.

1. 48 is what percent of 52?

2. 295 is what percent of 400?

3. What percent of 22 is 56?

4. What percent of 4 is 15?

5. What is 99% of 840?

6. What is 4.5% of 38?

7. What is 16% of 36.2?

8. 85 is 80% of what number?

9. 60 is 29% of what number?

10. 4.5 is 90% of what number?

7-3 Study Guide and Intervention *(continued)*

Using the Percent Proportion

Apply the Percent Proportion To apply the percent proportion to real-world problems, identify the numbers representing the part, whole, or percent relationship and use a variable for the missing information.

Example 1 **RESTAURANTS On Main Street in Jaime's town, there are 3 Mexican restaurants, 2 seafood restaurants, 4 fast-food restaurants, 2 Chinese restaurants, and 1 steak house restaurant. What percent of the restaurants on Main Street are Mexican restaurants?**

Compare the number of Mexican restaurants, 3, to the total number of restaurants, 12. The part is 3 and the whole is 12. Let p represent the percent.

$\frac{3}{12} = \frac{p}{100}$ Write the percent proportion.

$3 \cdot 100 = 12 \cdot p$ Find the cross products.

$300 = 12p$ Simplify.

$\frac{300}{12} = \frac{12p}{12}$ Divide each side by 12.

$p = 25$

So, 25% of the restaurants are Mexican restaurants.

Example 2 **COLLEGE In her freshman year of college, Caitlin took a total of 16 credits. Her math class was 25% of those credits. How many credits was her math class?**

Identify 16 as the total and 25 as the percent. Use a for the variable for the part of the credits that is her math class.

$\frac{a}{16} = \frac{25}{100}$ Write the percent proportion.

$a \cdot 100 = 16 \cdot 25$ Find the cross products.

$100a = 400$ Simplify.

$\frac{100a}{100} = \frac{400}{100}$ Divide each side by 100.

$a = 4$

So, Caitlin's math class was 4 credits.

Exercises

1. **PETS** At the pet store in the mall there are 21 dogs, 13 cats, 12 rabbits, 5 hamsters, and 3 ferrets. What percent of the animals in the pet store are rabbits?

2. **TIME** You spend 7 hours of your day at school. About what percent of the day do you spend at school?

3. **READING** You've read 234 pages of your book, which is about 78% of the book. How many pages are in the whole book?

NAME ______________________ DATE ____________ PERIOD ____________

7-4 Study Guide and Intervention

Find Percent of a Number Mentally

Find Percent of a Number Mentally When working with common percents like 10%, 25%, 40%, and 50%, it may be helpful to use the fraction form of the percent.

Percent-Fraction Equivalents				
$20\% = \frac{1}{5}$	$10\% = \frac{1}{10}$	$25\% = \frac{1}{4}$	$12\frac{1}{2}\% = \frac{1}{8}$	$16\frac{2}{3}\% = \frac{1}{6}$
$40\% = \frac{2}{5}$	$30\% = \frac{3}{10}$	$50\% = \frac{1}{2}$	$37\frac{1}{2}\% = \frac{3}{8}$	$33\frac{1}{3}\% = \frac{1}{3}$
$60\% = \frac{3}{5}$	$70\% = \frac{7}{10}$	$75\% = \frac{3}{4}$	$62\frac{1}{2}\% = \frac{5}{8}$	$66\frac{2}{3}\% = \frac{2}{3}$
$80\% = \frac{4}{5}$	$90\% = \frac{9}{10}$	$100\% = 1$	$87\frac{1}{2}\% = \frac{7}{8}$	$83\frac{1}{3}\% = \frac{5}{6}$

Example **Find 20% of 35 mentally.**

$20\% \text{ of } 35 = \frac{1}{5} \text{ of } 35$ Think: $20\% = \frac{1}{5}$.

$= 7$ Think: $\frac{1}{5}$ of 35 is 7. So, 20% of 35 is 7.

Exercises

Find the percent of each number mentally.

1. 50% of 6
2. 25% of 100
3. 60% of 25
4. 75% of 28
5. $66\frac{2}{3}\%$ of 33
6. 150% of 2
7. 125% of 4
8. 175% of 4
9. 10% of 110
10. 80% of 20
11. 20% of 80
12. 20% of 800
13. 30% of 250
14. 60% of 250
15. 75% of 1000
16. 10% of 900
17. 20% of 900
18. 40% of 900
19. 25% of 360
20. 50% of 360
21. 75% of 360
22. $62\frac{1}{2}\%$ of 32
23. $37\frac{1}{2}\%$ of 32
24. 200% of 21
25. $66\frac{2}{3}\%$ of 54
26. 150% of 2222
27. $12\frac{1}{2}\%$ of 720
28. 30% of 30
29. $66\frac{2}{3}\%$ of 150
30. 80% of 1500

7-4 Study Guide and Intervention *(continued)*

Find Percent of a Number Mentally

Estimate With Percents When an exact answer is not needed, estimate by rounding and using mental math to compute the answer.

Example **Estimate.**

a. 23% of 84

23% is about 25% or $\frac{1}{4}$.

$\frac{1}{4}$ of 84 is 21.

So, 23% of 84 is about 21.

b. $\frac{1}{2}$% of 490

$\frac{1}{2}\% = \frac{1}{2} \cdot 1\%$

490 is almost 500.

So, $\frac{1}{2}$% of 490 is about $\frac{1}{2} \times 5$ or 2.5.

c. 19% of 120

19% is about 20% of or $\frac{1}{5}$.

$\frac{1}{5}$ of 120 is 24.

So, 19% of 120 is about 24.

d. 180% of 15

100% of 15 is 15.

80% of 15 is 10.

So, 180% of 15 is about

15 + 12 or 27.

Exercises

Estimate.

1. 19% of 20

2. 52% of 129

3. 8% of 35

4. 72% of 12

5. $\frac{1}{2}$% of 390

6. 150% of 200

7. 33% of 33

8. 15% of 40

9. 22% of 310

10. 48% of 21

11. $\frac{5}{4}$% of 783

12. 119% of 510

13. 39% of 121

14. 53% of 695

15. 160% of 43

16. $\frac{1}{4}$% of 816

17. 27% of 16

18. 21% of 80

19. 130% of 9

20. $\frac{2}{3}$% of 602

7-5 Study Guide and Intervention

Using Percent Equations

Percent Equations A **percent equation** is an equivalent form of a percent proportion. In a percent equation, the percent is written as a decimal.

Example **Solve each problem using a percent equation.**

a. Find 22% of 95.

$n = 0.22(95)$

$n = 20.9$

So, 22% of 95 is 20.9.

b. 15 is what percent of 75?

$15 = n(75)$

$0.2 = n$

So, 15 is 20% of 75.

c. 90 is 20% of what number?

$90 = 0.2n$

$450 = n$

So, 90 is 20% of 450.

Exercises

Solve each problem using a percent equation.

1. Find 76% of 25.

2. Find 9% of 410.

3. Find 40% of 7.

4. Find 26% of 505.

5. Find 3.5% of 280.

6. Find 18.5% of 60.

7. Find 107% of 1080.

8. 256 is what percent of 800?

9. 36 is what percent of 240?

10. 2089.5 is what percent of 2100?

11. 15.4 is what percent of 55?

12. 7 is what percent of 350?

13. 13.2 is what percent of 80?

14. 14.4 is what percent of 120?

15. 36 is 9% of what number?

16. 2925 is 39% of what number?

17. 576 is 90% of what number?

18. 24.2 is 55% of what number?

19. 25 is 125% of what number?

20. 0.6 is 7.5% of what number?

7-5 Study Guide and Intervention *(continued)*

Using Percent Equations

Solve Problems The percent equation can be used to solve real-world problems.

Example **REAL ESTATE A commission is the fee paid to the real estate agent based on a percent of sales. If a real estate agent's commission is 3% and the house sold for $150,000, how much was the real estate agent's commission?**

The whole is $150,000. The percent is 3%. You need to find the amount of the commission, or the part. Let c represent the amount of the commission.

$$\underbrace{\text{part}} = \underbrace{\text{percent}} \cdot \underbrace{\text{whole}}$$

$c = 0.03 \cdot 150{,}000$ Write the percent equation, writing 3% as a decimal.

$c = 4500$ Multiply.

So, the real estate agent made $4500 in commission.

Exercises

1. **RUNNING** Emily is in training for a marathon. She ran 4 miles every day this week. She wants to increase her distance every week by 25%. How many miles a day will she run next week?

2. **TESTS** Juan got 15 questions correct on his pretest. He wants to get 20% more correct on his post test. How many questions does he want to get correct on his post test?

3. **CALORIES** The average person should eat around 2000 Calories a day. If Susan ate 1500 Calories, what percent of the average person's total did she eat?

4. **COMPUTERS** Chan bought a $600 computer, but his total was $648. What percent sales tax did he pay?

5. **JEANS** Jodi found a pair of jeans on sale for $90. Her friend told her that was only 75% of the original price. What was the original price of the jeans?

7-6 Study Guide and Intervention

Percent of Change

A **percent of change** tells how much an amount has increased or decreased in relation to the original amount. There are two methods you can use to find percent of change.

Example **Find the percent of change from 75 yards to 54 yards.**

Step 1 Subtract to find the amount of change.

$54 - 75 = -21$ final amount − original measurement

Step 2 Write a ratio that compares the amount of change to the original measurement. Express the ratio as a percent.

$$\text{percent of change} = \frac{\text{amount of change}}{\text{original measurement}}$$

$$= \frac{-21}{75} \quad \text{Substitution}$$

$$= -0.28 \text{ or } -28\,\% \quad \text{Write the decimal as a percent.}$$

Exercises

Find the percent of change. Round to the nearest tenth, if necessary. Then state whether the percent of change is an *increase* or *decrease*.

1. from 22 inches to 16 inches

2. from 8 years to 10 years

3. from \$815 to \$925

4. from 15 meters to 12 meters

5. from 55 people to 217 people

6. from 45 mi per gal to 24 mi per gal

7. from 28 cm to 32 cm

8. from 128 points to 144 points

9. from \$8 to \$2.50

10. from 800 roses to 639 roses

11. from 8 tons to 4.2 tons

12. from 5 qt to 18 qt

13. from \$85.75 to \$90.15

14. from 198 lb to 112 lb

7-6 Study Guide and Intervention *(continued)*

Percent of Change

Using Markup and Discount A store sells items for more than it pays for those items so it can make a profit. The amount of increase is called the **markup**. The percent of markup is a percent of increase. The amount the customer actually pays for an item is the **selling price**. When a store has a sale, the **discount** is the amount by which the regular price is reduced. The percent discount is a percent of decrease.

Example 1 **Find the selling price if a store pays $167 for a set of luggage and the markup is 38%.**

Method 1 Find the amount of the markup first.
The whole is $167. The percent is 38. You need to find the amount of the markup, or the part. Let m represent the amount of the markup.

$m = 0.38 \cdot 167$ part = percent • whole

$m = 63.46$ Multiply.

Add the markup to the cost. So, $167 + $63.46 = $230.46.

Method 2 Find the total percent first.
The customer will pay 100% of the store's price plus an extra 38%, or 138% of the store's price. Let p represent the price.

$p = 1.38(167)$ part = percent • whole

$p = 230.46$ Multiply.

The selling price is $230.46.

Example 2 **Find the sale price of a purebred German Shepherd puppy that is regularly $450 and is on sale for 35% off.**

Method 1 Find the amount of discount first. Let *d* represent the amount of the discount.

$d = 0.35 \cdot 450$ part = percent • whole

$d = 157.50$ Multiply.

Subtract the discount from the original cost. So, $450 – 157.50 = $292.50

Method 2 Find the total percent first. Let *p* represent the sale price.
The amount of the discount is 35%, so the customer will pay 100% – 35% or 65% of the original cost.

$p = 0.65(450)$ part = percent • whole

$p = 292.50$ Multiply.

The sale price is $292.50.

Exercises

Find the selling price for each item given the cost and the percent of markup.

1. guitar: $500; 60% markup

2. MP3 player: $28; 78% markup

3. lamp: $24; 18% markup

4. jeans: $26; 80% markup

5. MUSIC A record store is having a 25% off sale. Find the sale price of a CD that regularly costs $14.99.

7-7 Study Guide and Intervention

Simple Interest **Interest** is the amount of money paid or earned for the use of money by a bank or other financial institution. For a savings account, interest is earned. For a credit card, interest is paid. To solve problems involving interest, use the formula $I = prt$, where I is the interest, p is the principal (the amount of money invested or borrowed), r is the interest rate, and t is the time in years.

Example 1 **Find the simple interest for $500 invested at 3.2% for 5 years.**

$I = prt$	Write the simple interest formula.
$I = 500 \cdot 0.032 \cdot 5$	Replace *p* with 500, *r* with 0.032, and *t* with 5.
$I = 80$	Simplify.

The simple interest is $80.

Example 2 **REMODELING The Andersons borrowed $3000 to remodel their kitchen. They will pay $125 per month for 30 months. Find the simple interest rate for their loan.**

$\$125 \cdot 30 = \$3750$	Multiply to find the total amount paid back.
$\$3750 - \$3000 = \$750$	Subtract to find the interest.
$I = prt$	Write the simple interest formula.
$750 = 3000 \cdot r \cdot 2.5$	Replace *I* with 750, *p* with 3000, and *t* with 2.5 (30 months = 2.5 years).
$750 = 7500r$	Simplify.
$\frac{750}{7500} = \frac{7500r}{7500}$	Divide each side by 7500.
$0.1 = r$	The simple interest rate is 0.1 or 10%.

Exercises

Find the simple interest to the nearest cent.

1. $300 at 8% for 4 years

2. $1500 at 7.5% for 3 years

3. $1225 at 6.25% for 18 months

4. $900 at 12% for 60 months

5. $820 at 6% for 6 months

6. $13,000 at 13% for 2 years

7. CARS Cody borrowed $1500 to buy a used car. He will be paying back the money at a rate of 12% over the next 60 months. Find the amount of interest he will be paying on his loan.

8. SAVINGS Mr. and Mrs. Linden placed $12,000 in a certificate of deposit for 36 months for their son's college fund. At the end of that time, they earned $2160 in interest. What was the simple interest rate on the certificate of deposit?

9. LOANS Phoenix borrowed $20,000 to pay for her first year of college. She will be paying $225 every month for the next 10 years. What is the simple interest rate on her school loan?

7-7 Study Guide and Intervention

Compound Interest Simple interest is paid only on the initial principal of a savings account or a loan. **Compound interest** is paid on the initial principal and on interest earned in the past.

Example **What is the total amount of money in an account where $350 is invested at an interest rate of 7.25% compounded annually for 2 years?**

Step 1 Find the amount of money in the account at the end of the first year.

$I = prt$ Write the simple interest formula.

$I = 350 \cdot 0.0725 \cdot 1$ Replace *p* with 350, *r* with 0.0725, and *t* with 1.

$I = 25.375 \approx 25.38$ Simplify.

$350 + 25.38 = 375.38$ Add the amount invested and the interest.

At the end of the first year, there is $375.38 in the account.

Step 2 Find the amount of money in the account at the end of the second year.

$I = prt$ Write the simple interest formula.

$I = 375.38 \cdot 0.0725 \cdot 1$ Replace *p* with 375.38, *r* with 0.0725, and *t* with 1.

$I = 27.21505 \approx 27.22$ Simplify.

$375.38 + 27.22 = 402.60$ Add the amount invested and the interest.

At the end of the second year, there is $402.60 in the account.

Exercises

Find the total amount in each account to the nearest cent, if the interest is compounded annually.

1. $2825 at 4.75% for 2 years

2. $695 at 6.5% for 3 years

3. $18,000 at 13% for 3 years

4. $820 at 7% for 4 years

5. $530 at 5.5% for 5 years

6. $950 at 6.8% for 2 years

7. $640 at 8.2% for 3 years

8. $3500 at 11.9% for 4 years

7-8 Study Guide and Intervention

Circle Graphs

Circle Graphs A **circle graph** can be used to compare parts of a data set to the whole set of data. The percents in a circle graph add up to 100 because the entire circle represents the whole set.

Example **Construct a circle graph using the information in the table at the right.**

How Students Spend Time With Their Families	
Activity	**Number**
Dinner	60
TV	40
Talking	22
Sports	15
Walking	5
Other	20

Source: Scholastic

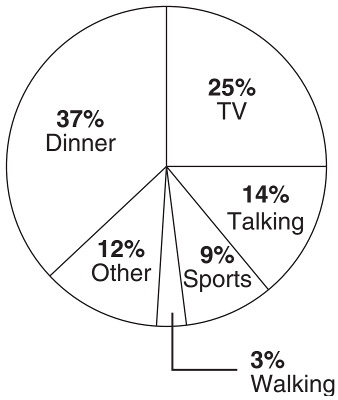
Step 1 Find the total number of students surveyed.
$60 + 40 + 22 + 15 + 5 + 20 = 162$

Step 2 Find the ratio that compares the number of students in each activity to the total number of students surveyed.
dinner: $60 \div 162 \approx 0.37$ sports: $15 \div 162 \approx 0.09$
TV: $40 \div 162 \approx 0.25$ walking: $5 \div 162 \approx 0.03$
talking: $22 \div 162 \approx 0.14$ other: $20 \div 162 \approx 0.12$

Step 3 There are 360° in a circle. So, multiply each ratio by 360 to find the number of degrees for each section of the graph.
dinner: $0.37 \cdot 360 \approx 133$ sports: $0.09 \cdot 360 \approx 32$
TV: $0.25 \cdot 360 = 90$ walking: $0.03 \cdot 360 \approx 11$
talking: $0.14 \cdot 360 \approx 51$ other: $0.12 \cdot 360 \approx 43$

Step 4 Use a compass to draw a circle and radius. Then use a protractor to draw a 90° angle. This section represents the number of students in the TV category. From the new radius, draw the next angle. Repeat for each of the remaining angles. Label each section. Then give the graph a title.

How Students Spend Time With Their Families



Exercise

Construct a circle graph for the following set of data.

1.

How Many Pets Do You Own?	
Pets	**Percent**
1	72
2	15
3	7
4	3
5	1
More than 5	2

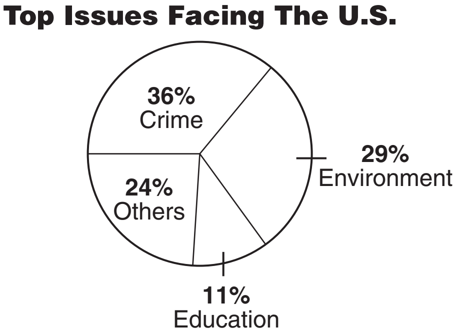
Source: PBS KIDS

7-8 Study Guide and Intervention *(continued)*

Circle Graphs

Analyze Circle Graphs Use the percents and central angle measures in a circle graph to solve real-world problems.

Example STUDENT OPINION **The circle graph at the right shows what issues students feel are the top issues facing the United States. Suppose 50,000 students were surveyed. How many more students feel the environment is more of a concern than education?**



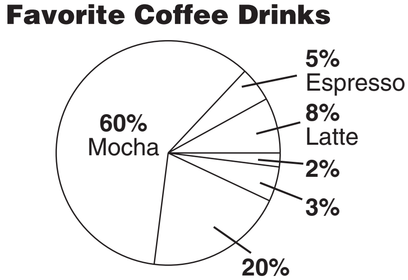
Environment: 29% of 50,000 = $0.29 \cdot 50{,}000$ or 14,500

Education: 11% of 50,000 = $0.11 \cdot 50{,}000$ or 5500

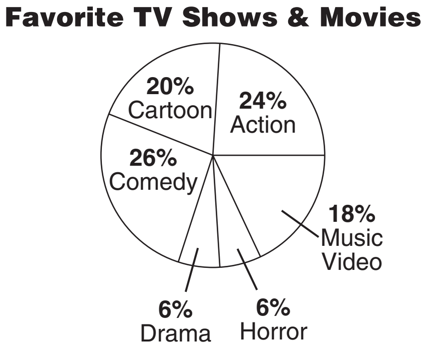
So, 14,500 – 5500 or 9000 students feel the environment is more of a concern than education.

Exercises

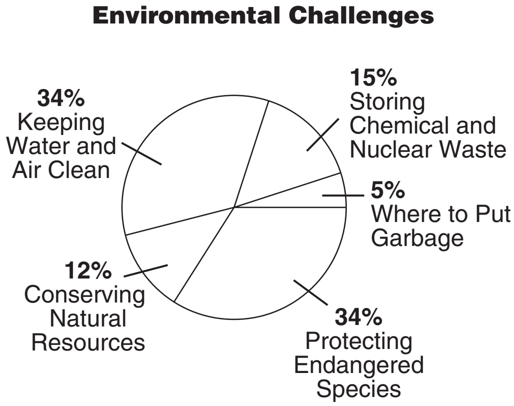
1. **COFFEE** The circle graph at the right shows the results of a survey about favorite kinds of coffee. If 500 people were surveyed, how many more people like mochas better than lattes?



2. **ENTERTAINMENT** The circle graph at the right shows the results of a survey about favorite kinds of television shows and movies kids prefer. If 5,564 kids were surveyed, how many more preferred cartoons to horror?



3. **ENVIRONMENT** The circle graph at the right shows the results of a survey about what students feel will be the biggest environmental challenge of the 21st century. If 834 students were surveyed, how many fewer students think the concern for where to put the garbage will not be as challenging as where to store chemicals and nuclear waste?



8-1 Study Guide and Intervention

Functions

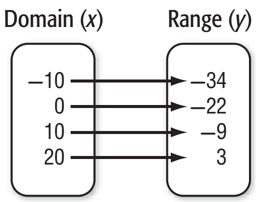
Relations and Functions A function is a special relation in which each element of the domain is paired with exactly one element of the range. To determine whether a relation is a function, list the domain and range of the relation and make sure that each member of the domain pairs up with only one value in the range. Another method is to apply the **vertical line test** to the graph of the relation.

Vertical Line Test	Move a pencil or straightedge from left to right across the graph of a relation. • If it passes through no more than one point on the graph, the graph represents a function. • If it passes through more than one point on the graph, the graph does not represent a function.

Since functions are relations, they can be represented using ordered pairs, tables, or graphs.

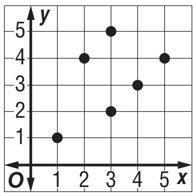
Example **Determine whether each relation is a function. Explain.**

a. **{(– 10, –34), (0, –22), (10, –9), (20, 3)}**



Because each element in the domain is paired with only one value in the range, this is a function.

b.



Use a pencil or straightedge and move from left to right across the graph. It passes through more than one point of the graphed relation at $x = 3$. Therefore, this is not a function.

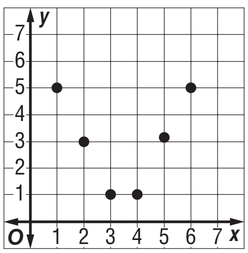
Exercises

Determine whether each relation is a function. Explain.

1. {(–5, 2), (3, –3), (1, 7), (3, 0)}

2. {(2, 7), (–5, 20), (–10, 20), (–2, 10), (1, 20)}

3.



4.

x	8	1	–5	1	–10
y	–2	3	7	7	13

8-1 Study Guide and Intervention *(continued)*

Functions

Function Notation Functions that can be written as equations can be written in **function notation**, where the variable y and the term $f(x)$ represent the dependent variable. The term $f(x)$ is read "f at x."

equation	function notation
$y = 5x - 2$	$f(x) = 5x - 2$

Example 1 **If $f(x) = 3x + 4$, find the function value for $f(-2)$.**

$f(x) = 3x + 4$ Write the function.

$f(-2) = 3(-2) + 4$ or -2 Substitute –2 for x into the function rule.

So, $f(-2) = -2$.

Example 2 **DRIVING Janie drove 210 miles at 42 miles per hour. Use function notation to write an equation that gives the total mileage as a function of the number of hours driven. Then use the equation to determine the number of hours Janie drove.**

First write the equation.

Words: miles driven = miles per hour times the number of hours

Variables: Let $m(h)$ = miles driven and h = number of hours.

Function: $m(h) = 42 \cdot h$

The function is $m(h) = 42h$.

Next, use the equation to find how many hours Janie drove.

$m(h) = 42h$ Write the function.

$210 = 42h$ Substitute 210 for $m(h)$.

$5 = h$ Divide each side by 42.

So, Janie drove for 5 hours.

Exercises

If $f(x) = -3x + 2$, find each function value.

1. $f(9)$
2. $f(12)$
3. $f(-2)$
4. $f(-5)$
5. $f(13)$
6. $f(-25)$
7. $f(300)$
8. $f(-150)$

If $f(x) = 5x - 6$, find each function value.

9. $f(8)$
10. $f(-12)$
11. $f(3)$
12. $f(-1)$
13. $f(30)$
14. $f(-14)$
15. $f(-9)$
16. $f(70)$

17. **PHONES** Charlene's phone service costs $14 a month plus $0.20 per minute. Last month, her phone bill was $44. Use function notation to write an equation that gives the total cost as a function of the number of minutes used. Then use the equation to find how many minutes Charlene used.

8-2 Study Guide and Intervention

Sequences and Equations

Describe Sequences A **sequence** is an ordered list of numbers. Each number is a **term** of the sequence. An **arithmetic sequence** is a sequence in which the difference between any two consecutive terms, called the **common difference**, is the same.

In the sequence below, the common difference is 7.

2, 9, 16, 23, 30, . . .,

+7 +7 +7 +7

Example **Describe the sequence using words and symbols.**

3, 6, 9, 12, …

+1 +1 +1

Term Number *(n)*	1	2	3	4
Term *(t)*	3	6	9	12

+3 +3 +3

The difference of the term numbers is 1.
The common difference of the terms is 3.

The terms have a common difference of 3. A term is 3 times the term number. So, the equation that describes the sequence is $t = 3n$.

Exercises

Describe each sequence using words and symbols.

1. 4, 5, 6, 7, …

2. 6, 7, 8, 9, …

3. 6, 12, 18, 24, …

4. 8, 16, 24, 32, …

5. 4, 8, 12, 16, …

6. 9, 18, 27, 36, …

7. 11, 22, 33, 44, …

8. 3, 7, 11, 15, …

9. 6, 8, 10, 12, …

8-2 Study Guide and Intervention *(continued)*

Sequences and Equations

Finding Terms The rule or equation that describes a sequence can be used to either extend the pattern or to find other terms.

Example 1 **Write an equation that describes the sequence 5, 7, 9, 11, …. Then find the 14th term of the sequence.**

Term Number *(n)*	1	2	3	4
Term *(t)*	5	7	9	11

+1 +1 +1

+2 +2 +2

The difference of the term numbers is 1.

The common difference of the terms is 2.

The terms have a common difference of 2. This is 2 times the difference of the term numbers. This suggests that $t = 2n$. However, you need to add 3 to get the value of t. So, a term is 3 more than 2 times the term number.

The equation that describes the sequence is $t = 2n + 3$.

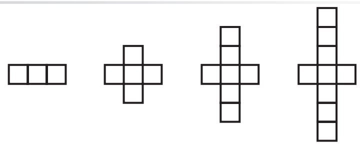
Use the equation to find the 14th term. Let $n = 14$.

$t = 2n + 3$ Write the equation.

$t = 2(14) + 3$ or 31 Replace n with 14.

So, the 14th term is 31.

Example 2 **Daisy made the figures shown at the right with tiles. Each tile has an area of 1 square foot. If she continues the pattern, which figure would have an area of 25 square feet?**



1 2 3 4

The difference of the term numbers is 1.

The common difference of the terms is 2.

Make a table to organize your sequence and find a rule.

Term Number *(t)*	1	2	3	4
Term *(a)*	3	5	7	9

The pattern in the table shows the equation $a = 2t + 1$.

$a = 2t + 1$ Write the equation.

$25 = 2t + 1$ Replace a with 25. Solve for t.

So, figure 12 would be a design with an area of 25 square feet.

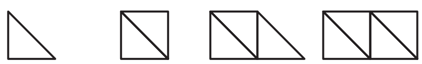
Exercises

Write an equation that describes each sequence. Then find the indicated term.

1. 5, 13, 21, 29, …; 11th term

2. 6, 10, 14, 18, …; 14th term

3. GEOMETRY Rhonda made the figures shown at the right using toothpicks. Each toothpick has a length of 1 inch. If she continues the pattern, which figure will have a perimeter of 25 inches?



1 2 3 4

NAME ______________________ DATE __________ PERIOD __________

8-3 Study Guide and Intervention

Representing Linear Functions

Solve Linear Equations An equation whose graph is a line is called a **linear equation**. Examples of linear equations are given below.

$y = 3x$	$y = x + 7$	$y = \frac{x}{5}$	$y = 8 - 2x$

A linear equation is also a function because each member of the domain (x-value) is paired with exactly one member of the range (y-value). Solutions to a linear equation are ordered pairs that make the equation true. One way to find solutions to an equation is to make a table.

Example **Find four solutions of $y = 4x - 10$. Write the solutions as ordered pairs.**

Step 1 Choose four values for x and substitute each value into the equation. We choose −1, 0, 1, and 2.

Step 2 Evaluate the expression to find the value of y.

Step 3 Write the solutions as ordered pairs.

x	y = 4x − 10	y	(x, y)
−1	$y = 4(-1) - 10$	−14	(−1, −14)
0	$y = 4(0) - 10$	−10	(0, −10)
1	$y = 4(1) - 10$	−6	(1, −6)
2	$y = 4(2) - 10$	−2	(2, −2)

Four solutions of $y = 4x - 10$ are (−1, −14), (0, −10), (1, −6), and (2, −2).

Exercises

Copy and complete each table. Use the results to write four ordered pair solutions of the given equation.

1. $y = x + 2$

x	y = x + 2	y
−2		
0		
2		
4		

2. $y = 5x - 6$

x	y = 5x − 6	y
−1		
0		
1		
2		

Find four solutions of each equation. Write the solutions as ordered pairs.

3. $y = 9 - x$

4. $y = x + 12$

5. $y = x - 7$

6. $y = 2x + 4$

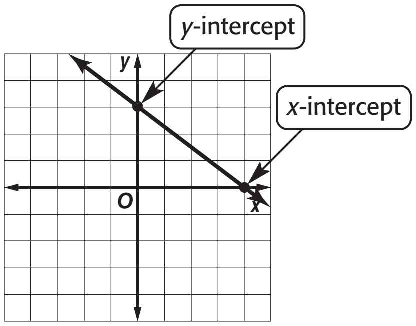
7. $y = -3x - 7$

8. $4x + y = 5$

8-3 Study Guide and Intervention *(continued)*

Representing Linear Functions

Graph Linear Equations You can plot points on a coordinate plane to graph a linear equation. You can find ordered pairs using a table, or you can plot the x-intercept and the y-intercept and connect the two points. The x-intercept is the x-coordinate of the point at which the graph crosses the x-axis. The y-intercept is the y-coordinate of the point at which the graph crosses the y-axis.



Example **Graph $2x + y = 6$.**

You can graph an equation by using a table to find ordered pairs.

Step 1 Rewrite the equation by solving for y.

$2x + y = 6$ Write the equation.

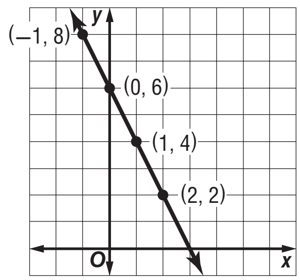
$2x - 2x + y = 6 - 2x$ Subtract $2x$ from each side.

$y = 6 - 2x$ Simplify.

x	y = 6 – 2x	y	(x, y)
–1	y = 6 – 2(–1)	8	(–1, 8)
0	y = 6 – 2(0)	6	(0, 6)
1	y = 6 – 2(1)	4	(1, 4)
2	y = 6 – 2(2)	2	(2, 2)

Step 2 Choose four values for x and find the corresponding values for y. Four solutions are (–1, 8), (0, 6), (1, 4) and (2, 2).

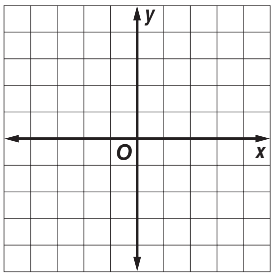
Step 3 Graph the ordered pairs on a coordinate plane and draw a line through the points.



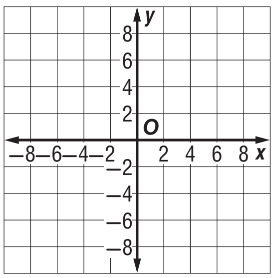
Exercises

Graph each equation by plotting ordered pairs.

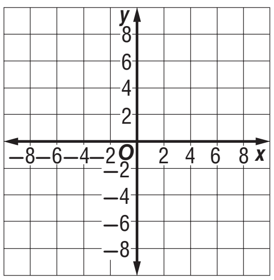
1. $y = -4x$



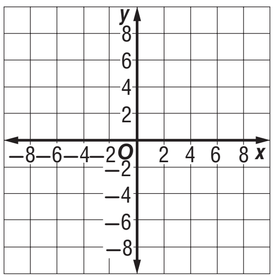
2. $y = x + 6$



3. $x + y = -4$



4. $-4x + y = -3$

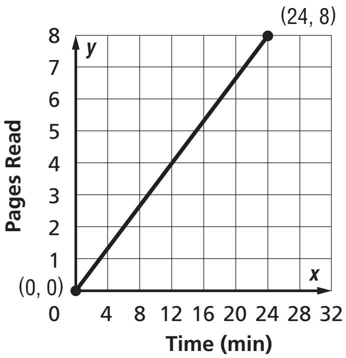


8-4 Study Guide and Intervention

Rate of Change

Rate of Change A **rate of change** is a rate that describes how one quantity changes in relation to another quantity.

Example 1 **Find the rate of change for the linear function represented in the graph.**



$$\text{rate of change} = \frac{\text{change in pages read}}{\text{change in time}}$$

$$= \frac{8 \text{ pages} - 0 \text{ pages}}{24 \text{ minutes} - 0 \text{ minutes}}$$

$$= \frac{8 \text{ pages}}{24 \text{ minutes}}$$

$$= \frac{1}{3} \text{ page per minute}$$

So, the rate of change is $\frac{1}{3}$ page/minute, or an increase of $\frac{1}{3}$ page per 1 minute increase in time.

Example 2 **Find the rate of change for the linear function represented in the table.**

$$\text{rate of change} = \frac{\text{change in temperature}}{\text{change in time}}$$

$$= \frac{2}{1} \text{ or } 2$$

Time (h)	x	0	1	2	3
Temperature (˚C)	y	0	2	4	6

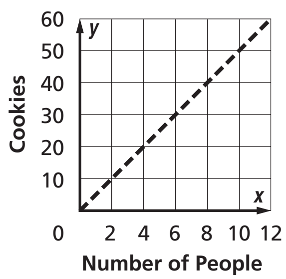
Exercises

Find the rate of change for each linear function.

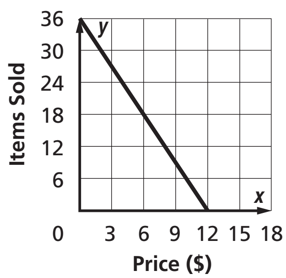
1.

Time (h)	x	0	2	4	6
Distance Flown (mi)	y	0	1000	2000	3000

2. **Cookies Needed**



3. **Sales**



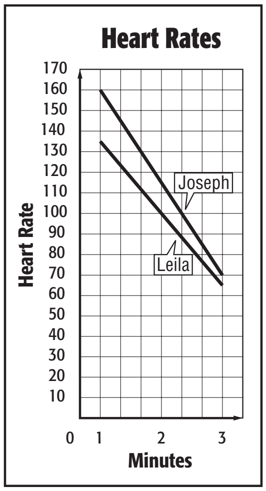
8-4 Study Guide and Intervention *(continued)*

Rate of Change

Interpret Rates of Change

You can calculate rates of change to solve problems.

Example **EXERCISE The graph shows Leila and Joseph's heart rates during the 3 minutes after they exercised. Compare the rates of change.**



Leila's Heart Rate:

$$\text{rate of change} = \frac{\text{change in } y}{\text{change in } x}$$

$$= \frac{135 - 65}{3 - 1} = \frac{70}{2}$$

So, the rate of change is 35 beats per minute.

Joseph's Heart Rate:

$$\text{rate of change} = \frac{\text{change in } y}{\text{change in } x}$$

$$= \frac{160 - 70}{3 - 1} = \frac{90}{2}$$

So, the rate of change is 45 beats per minute.

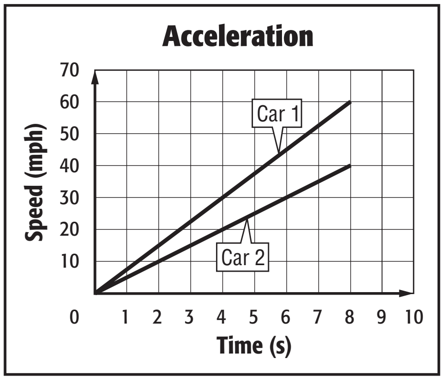
Joseph's heartbeat decreased at a greater rate than Leila's heartbeat.

Exercises

1. SCIENCE Ned boiled 2 beakers of water. He put beaker 1 in a pot of cold water to cool. The table shows the temperature in the two beakers. Compare the rates of change.

Time (m)	Temperature (°C)	
	Beaker 1	Beaker 2
0	100	100
1	96	100
2	92	99
3	88	99

2. CARS The graph at the right shows the speed at which two cars accelerated from 0 miles per hour. Compare the rates of change.



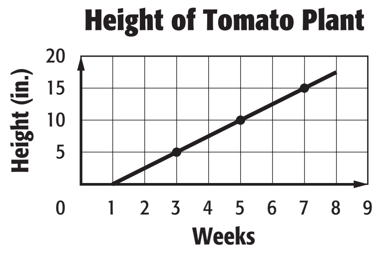
NAME ______________________________ DATE ____________ PERIOD ____________

8-5 Study Guide and Intervention

Constant Rate of Change and Direct Variation

Constant Rates of Change Relationships that have a straight-line graph are called **linear relationships**. A linear relationship has a **constant rate of change**, which means that the rates of change between any two data points is the same. In a given linear relationship, if the ratio of each non-zero y-value to the corresponding x-value is the same, the linear relationship is also **proportional**.

Example **GARDENS Gina recorded the height of a tomato plant in her garden. Find the constant rate of change for the plant's growth in the graph shown. Describe what the rate means. Then determine whether there is a proportional linear relationship between the plant height and the time.**



Step 1 Choose any two points on the line, such as (3, 5) and (7, 15).

(3, 5) 3 weeks, height 5 in.

(7, 15) 7 weeks, height 15 in.

Step 2 Find the rate of change between the points.

$$\text{rate of change} = \frac{\text{change in height}}{\text{change in time}}$$

$$= \frac{1.5 \text{ in.} - 5 \text{ in.}}{7 \text{ wk} - 3 \text{ wk}} = \frac{10 \text{ in.}}{4 \text{ wk}}$$ The height goes from 5 in. to 15 in. The time goes from 3 weeks to 7 weeks.

$$= 2.5 \text{ in./wk}$$ Express this as a unit rate.

The rate of change 2.5 in./wk means the plant is growing at a rate of 2.5 inches per week.

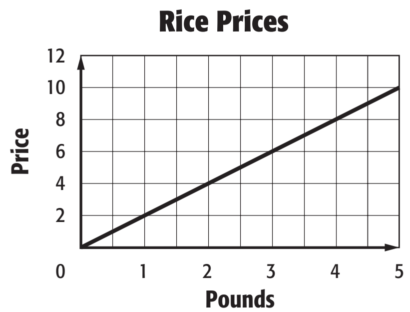
To determine if the quantities are proportional, find the $\frac{\text{height } y}{\text{time } x}$ for points on the graph.

$\frac{5 \text{ in.}}{3 \text{ wk}} \approx 1.67$ in./wk $\frac{10 \text{ in.}}{3 \text{ wk}} = 2$ in./wk $\frac{15 \text{ in.}}{7 \text{ wk}} \approx 2.14$ in./wk

The ratios are not equal, so the linear relationship is not proportional.

Exercise

1. Find the constant rate of change for the linear function at the right and interpret its meaning. Then determine whether a proportional linear relationship exists between the two quantities. Explain your reasoning.



8-5 Study Guide and Intervention *(continued)*

Constant Rate of Change and Direct Variation

Direct Variation When the ratio of two variable quantities is constant, their relationship change is called a **direct variation**. The graph of a direct variation always passes through the origin and can be expressed as $y = kx$, where k is called the **constant of variation**, or **constant of proportionality**.

Example **SCUBA DIVING As scuba divers descend below the surface of the ocean, the pressure that they feel from the water varies directly with the depth.**

Depth (ft)	Water Pressure (lb/in^2)
x	y
20	8.9
30	13.35
40	17.8
50	22.25

a. Write an equation that relates the depth and the amount of water pressure.

Step 1 Find the value of k using the equation $y = kx$. Choose any point in the table. Then solve for k.

$y = kx$ Direct variation equation

$17.8 = k(40)$ Replace y with 17.8 and x with 40.

$0.445 = k$ Simplify.

Step 2 Use k to write an equation.

$y = kx$ Direct variation equation

$y = 0.445x$ Replace k with 0.445.

b. Predict what the pressure will be at 28 feet.

$y = 0.445x$ Write the direct variation equation.

$y = 0.445(28)$ Replace x with 28.

$y = 12.46$ Simplify.

The depth at 28 feet will be 12.46 lb/in^2.

Exercises

1. **MONEY** The amount that Jared earns every week varies directly with the number of hours that he works. Suppose that last week he earned $75 for 6 hours of work. Write an equation that could be used to find how much Jared earns per hour. Then find out how much Jared would earn if he worked 25 hours.

2. **GASOLINE** The cost of buying gas varies directly with the number of gallons purchased. Suppose that Lena bought 12.2 gallons of gas for $35.99. Write an equation that could be used to find the unit cost per gallon of gas. Then find out how much 9.5 gallons of gas would cost. Round to the nearest cent.

3. **GEOMETRY** The circumference of a circle is in direct variation with the diameter of the circle. Kwan drew a circle with a circumference of 47.1 inches and a diameter of 15 inches. Write an equation that relates the circumference to the diameter. Use the equation to find the circumference of a circle with a 12-inch diameter.

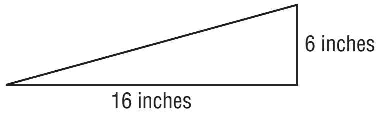
8-6 Study Guide and Intervention

Slope

Slope **Slope** describes the steepness of a line. It is the ratio of the rise, or vertical change, to the run, or horizontal change, of a line.

slope = $\frac{\text{rise}}{\text{run}}$ ← vertical change
← horizontal change

Example 1 **MODEL CARS Find the slope of a ramp designed to race model cars that rises 6 inches for every horizontal change of 16 inches.**



slope = $\frac{\text{rise}}{\text{run}}$ Write the formula for slope.

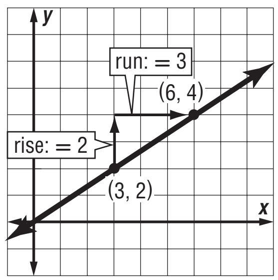
= $\frac{6 \text{ in.}}{16 \text{ in.}}$ rise = 6 in., run = 16 in.

= $\frac{3}{8}$ Simplify.

The slope of the ramp is $\frac{3}{8}$ or 0.375.

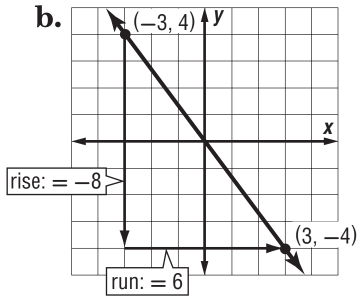
Example 2 **Find the slope of each line.**

a.



slope = $\frac{\text{rise}}{\text{run}} = \frac{2}{3}$

b.



slope = $\frac{\text{rise}}{\text{run}} = -\frac{8}{6}$ or $-\frac{4}{3}$

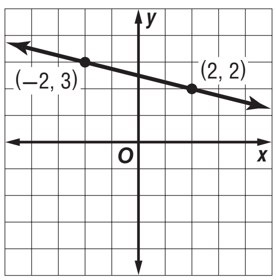
Exercises

1. What is the slope of a hill that rises 3 feet for every horizontal change of 12 feet? Write as a fraction in simplest form.

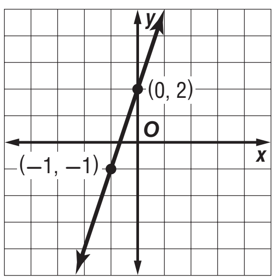
2. Mr. Watson is building a staircase. What is the slope of the staircase if it rises 20 inches for every horizontal change of 25 inches? Write as a fraction in simplest form.

Find the slope of each line.

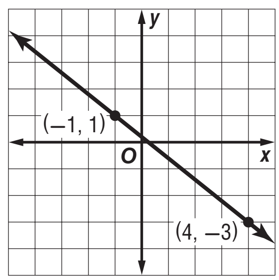
3.



4.



5.



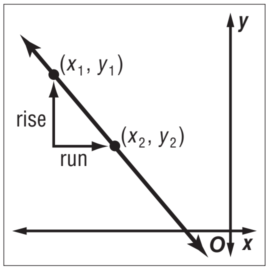
8-6 Study Guide and Intervention *(continued)*

Slope

Slope and Constant Rate of Change

Note that the slope is the same for any two points on a straight line. It represents a constant rate of change.

Model



Words The slope m of a line passing through points (x_1, y_1) and (x_2, y_2) is the ratio of the difference in the y-coordinates to the corresponding difference in x-coordinates.

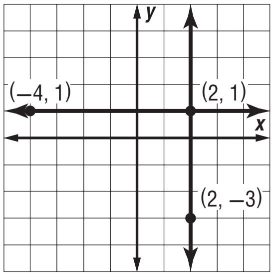
Symbols $m_1 = \dfrac{y_2 - y_1}{x_2 - x_1}$, where $x_2 \neq x_1$

Horizontal lines have a slope of 0.

$m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{1 - 1}{2 - (-4)} = \dfrac{0}{6}$ or 0.

Vertical lines have an undefined slope.

$m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{1 - (-3)}{2 - 2} = \dfrac{4}{0}$ Division by 0 is undefined.



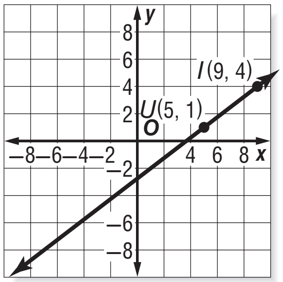
Example **Find the slope of the line that passes through *I*(9, 4) and *U*(5, 1).**

$m = \dfrac{y_2 - y_1}{x_2 - x_1}$ Definition of slope

$m = \dfrac{4 - 1}{9 - 5}$ $(x_1, y_1) = (5, 1)$, $(x_2, y_2) = (9, 4)$

$m = \dfrac{3}{4}$ Simplify.

The slope is $\dfrac{3}{4}$.



Exercises

Find the slope of the line that passes through each pair of points.

1. *A*(2, 2), *B*(−5, 4) **2.** *L*(5, 5), *M*(4, 2) **3.** *R*(7, −4), *S*(7, 3)

4. *Q*(3, 9), *R*(−5, 3) **5.** *C*(−4, 0), *D*(12, 2) **6.** *S*(−8, −2), *T*(1, 4)

7. *G*(5, 7), *H*(2, 7) **8.** *D*(2, 5), *E*(−6, −3) **9.** *K*(0, −3), *L*(−4, 2)

8-7 Study Guide and Intervention

Slope-Intercept Form

Find Slope and y-intercept An equation with a y-intercept that is *not* 0 represents a non-proportional relationship. An equation of the form $y = mx + b$, where m is the slope and b is the y-intercept, is also in slope-intercept form.

Example 1 **State the slope and the y-intercept of the graph of $y = -\frac{2}{3}x - 0.5$.**

$y = -\frac{2}{3}x - 0.5$ Write the equation.

$y = -\frac{2}{3}x + (-0.5)$ Write the equation in the form $y = mx + b$.

$y = mx + b$ $m = -\frac{2}{3}$, $b = -0.5$

The slope is $-\frac{2}{3}$ and the y-intercept is -0.5.

Example 2 **State the slope and the y-intercept of the graph of $6x - y = 7$.**

Write the equation in slope-intercept form.

$6x - y = 7$ Write the original equation.

$-6x \quad -6x$ Subtract $6x$ from each side.

$-y = 7 - 6x$ Simplify.

$-y = -6x + 7$ Write in slope-intercept form. Divide both sides by –1 to remove the negative

$y = 6x - 7$ coefficient from y.

$y = mx + b$ $m = 6$, $b = -7$

The slope of the graph is 6 and the y-intercept is -7.

Exercises

State the slope and the y-intercept of the graph of each equation.

1. $y = 4x + 12$

2. $y = -2x - 1$

3. $y = -x + 4$

4. $y = x - 9$

5. $y = \frac{5}{6}x - 8$

6. $5x - y = 22$

7. $3x + y = 8$

8. $y - x = 17$

9. $12x = y - 9$

10. $-3x = y + 1$

11. $y + 9x = 11$

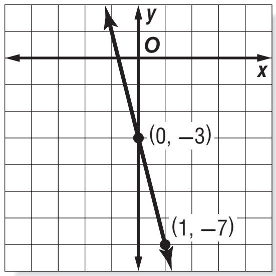
12. $y - 8x = 21$

8-7 Study Guide and Intervention *(continued)*

Slope-Intercept Form

Graph Equations Equations written in the slope-intercept form can be easily graphed.

Example **Graph $y = -4x - 3$ using the slope and y-intercept.**



Step 1 Find the slope and y-intercept.

slope = -4

y-intercept = -3

Step 2 Graph the y-intercept point at $(0, -3)$.

Step 3 Write the slope as $\frac{-4}{1}$. Use it to locate a second point on the line.

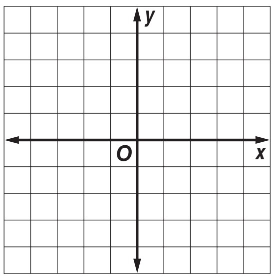
$m = \frac{-4}{1}$ ← change in y: down 4 units; ← change in x: right 1 unit

Step 4 Draw a line through the two points and extend the line.

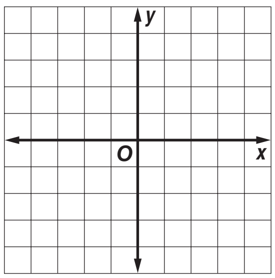
Exercises

Graph each equation using slope and y-intercept.

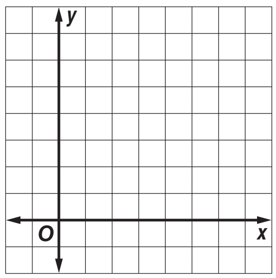
1. $y = 4x - 1$



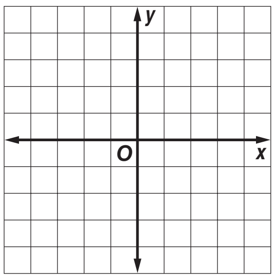
2. $y = 6x + 4$



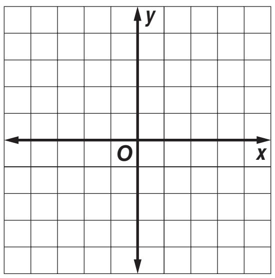
3. $y = \frac{1}{4}x + 5$



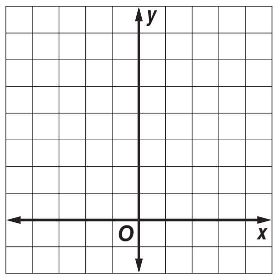
4. $y = 3x - 2$



5. $y = \frac{2}{3}x + 3$



6. $y = 5x - 3$



8-8 Study Guide and Intervention

Writing Linear Equations

Write Equations in Slope-Intercept Form If you know the slope and y-intercept, you can write the equation of a line by substituting these values in $y = mx + b$.

Example 1 **Write an equation in slope-intercept form for each line.**

a. slope = $-\frac{1}{4}$, y-intercept = –3

$y = mx + b$	Slope-intercept form
$y = -\frac{1}{4}x + (-3)$	Replace m with $-\frac{1}{4}$ and b with -3.
$y = -\frac{1}{4}x - 3$	Simplify.

b. slope = 0, y-intercept = –9

$y = mx + b$	Slope-intercept form
$y = 0x + (-9)$	Replace m with 0 and b with -9.
$y = -9$	Simplify.

An equation in the form $y - y_1 = m(x - x_1)$ where m represents the slope and (x_1, y_1) represents a point on the line is called **point-slope form** of a line.

Example 2 **Write an equation for the line that passes through (–4, 4) and (2, 7).**

Step 1 Find the slope m.

$m = \frac{y_2 - y_1}{x_2 - x_1}$	Definition of slope
$m = \frac{7 - 4}{2 - (-4)}$ or $\frac{1}{2}$	$(x_1, y_1) = (-4, 4)$, $(x_2, y_2) = (2, 7)$

Step 2 Use the slope and the coordinates of either point to write the equation in point-slope form.

$y - y_1 = m(x - x_1)$	Point-slope form
$y - 4 = \frac{1}{2}(x + 4)$	Replace (x, y) with $(-4, 4)$ and m with $\frac{1}{2}$.

The equation in point-slope form is $y - 4 = \frac{1}{2}(x + 4)$.

The equation in slope-intercept form is $y = \frac{1}{2}x + 6$.

Exercises

Write an equation in slope-intercept form for each line.

1. slope = 1, y-intercept = 2

2. slope = $-\frac{3}{4}$, y-intercept = –5

3. slope = 0, y-intercept = –3

Write an equation for the line in slope-intercept form that passes through each pair of points.

4. (6, 2) and (3, 1)

5. (8, 8) and (–4, 5)

6. (7, –3) and (–5, –3)

8-8 Study Guide and Intervention *(continued)*

Writing Linear Equations

Solve Problems Once you write an equation to describe the relationship between two quantities, you can use the equation to make predictions.

Example **A video game Web site charges a registration fee plus a monthly fee. After 2 months, the total fee is $34.90. After 6 months, the total fee is $74.70. What would be the total fee after 10 months?**

Understand You know the total fees at 2 months and 6 months. You need to find the total fee after 10 months.

Plan First, find the slope and the y-intercept. Then write an equation to show the relationship between the number of months x and the total fee y. Use the equation to find the total fee.

Solve Find the slope m.

$m = \frac{\text{change in } y}{\text{change in } x}$ ← change in fee / ← change in months

$= \frac{74.70 - 34.90}{6 - 2}$ or 9.95

Use the slope and the coordinates of either point to write the equation in point-slope form.

$y - y_1 = m(x - x_1)$ Point-slope form

$y - 74.7 = 9.95(x - 6)$ Replace (x_1, y_1) with (6, 74.4) and m with 9.95.

$y = 9.95x - 59.7 + 74.7$

The equation of the line in slope-intercept form that passes through (2, 34.9) and (6, 74.7) is $y = 9.95x + 15$.

Find the total fee.

$y = 9.95x + 15$ Write the equation.

$y = 9.95(10) + 15$ Replace x with 10.

$y = 114.5$ Simplify.

After 10 months, the total fee would be $114.50.

Exercises

1. **HEALTH CLUBS** A health club has a monthly membership with an initial registration fee. After 6 months, the total cost is $285, and after 9 months it is $390. Write an equation in slope-intercept form to represent the data. Describe what the slope and intercept mean. Use the equation to find the total fee after 15 months.

2. **MOVIES** A local movie theater has a movie lovers club. After paying a membership fee, all ticket purchases are discounted. The cost after buying 5 movie tickets is $48.75. The cost after buying 7 movie tickets is $58.25. Write an equation in slope-intercept form to represent the data. Describe what the slope and intercept mean. Use the equation to find the total cost after buying 12 tickets.

8-9 Study Guide and Intervention

Prediction Equations

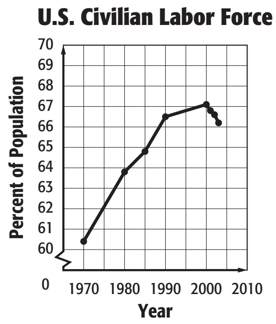
Line of Fit The graphs of real-life data often do not form a straight line. However, they may be close to a linear relationship. A **line of fit** is a line that is very close to most of the data points.

Example **The table shows the percent of the population in the U.S. labor force.**

a. Make a scatter plot and draw a line of fit for the data.

Year	Percent of Population	Year	Percent of Population
1970	60.4	2000	67.1
1980	63.8	2001	66.8
1985	64.8	2002	66.6
1990	66.5	2003	66.2

Source: U.S. Census Bureau



Source: U.S. Census Bureau

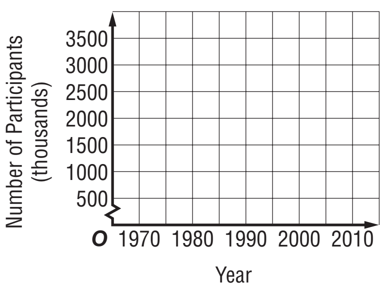
b. Use the line of fit to predict the percent of the population in the U.S. labor force in 2010.

Extend the line to find the y-value for an x-value of 2010. The corresponding y-value for the x-value of 2010 is about 70. So, about 70% of the U.S. population will be in the labor force in 2010.

Exercise

1. **Use the table that shows the number of girls who participated in high school athletic programs in the United States from 1973 to 2003.**

   a. Make a scatter plot and draw a line of fit.



Year	1973	1978	1983	1988	1993	1998	2003
Number of Participants (thousands)	817	2083	1780	1850	1997	2570	2856

Source: U.S. Census Bureau

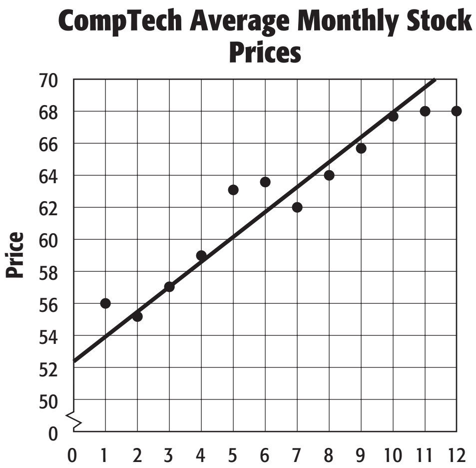
   b. Use the line of fit to predict the number of female participants in 2010.

8-9 Study Guide and Intervention *(continued)*

Prediction Equations

Prediction Equations Predictions about real-life data can also be made from the equation of the line of fit.

Example **STOCKS The scatter plot shows the average monthly price of CompTech's stocks.**



a. Write an equation in slope-intercept form for the line of fit that is drawn.

Step 1 Use two points on the line to find the slope. These may or may not be original data points.

$m = \frac{y_2 - y_1}{x_2 - x_1}$ — Definition of slope

$m = \frac{66 - 57}{9 - 3}$ — Use $(x_1, y_1) = (3, 57)$ and $(x_2, y_2) = (9, 66)$.

$m = 1.5$ — Simplify.

Step 2 Use the slope and the coordinates of either point to write the equation of the line in point-slope form.

$y - y_1 = m(x - x_1)$ — Point-slope form

$y - 66 = 1.5(x - 9)$ — Replace (x_1, y_1) with (9, 66) and m with 1.5.

Step 3 Solve the point-slope equation for y.

$y - 66 = 1.5(x - 9)$ — Point-slope equation

$y - 66 = 1.5x - 13.5$ — Distributive Property

$\quad + 66 \qquad + 66$ — Add 66 to each side.

$y = 1.5x + 52.5$ — Simplify.

The equation for the line of fit is $y = 1.5x + 52.5$.

b. Predict the stock price for month 15.

$y = 1.5x + 52.5$ — Write the equation of the line of fit.

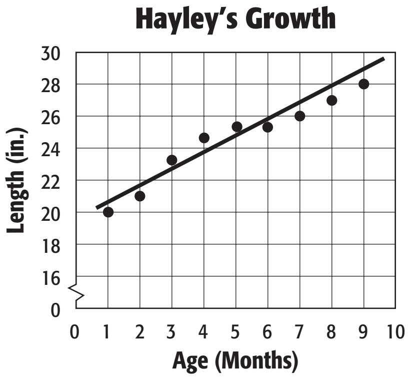
$y = 1.5(15) + 52.5$ — Replace x with 15.

$y = 75$ — Simplify.

A prediction of the stock price for month 15 is $75.

Exercise

1. **HEALTH** The scatter plot shows a baby's growth over 9 months.
   a. Write an equation in slope-intercept form for the line of fit that is drawn.
   b. Predict the baby's length at 12 months.



8-10 Study Guide and Intervention

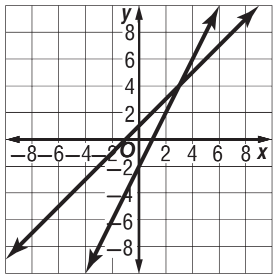
Systems of Equations

Solve Systems by Graphing A collection of two or more equations with the same set of variables is a **system of equations**. The solution to a system of equations with two variables, x and y, are the coordinate pair (x, y). If you graph both equations on the same coordinate plane, the coordinates of the point of intersection are the solution.

Example 1 **Solve the system of equations by graphing.**

$y = x + 1$

$y = 2x - 2$



The graphs appear to intersect at (3, 4). Check this estimate by substituting the coordinates into each equation.

Check	$y \stackrel{?}{=} x + 1$	$y = 2x - 2$
	$4 \stackrel{?}{=} 3 + 1$	$4 \stackrel{?}{=} 2(3) - 2$
	$4 = 4$ ✓	$4 = 4$ ✓

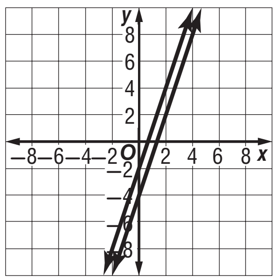
The solution of the system of equations is (3, 4).

> Systems of equations can have one solution, no solution, or infinitely many solutions. When the graphs of a system of equation are
> - parallel lines, there are no solutions.
> - the same graph, there are infinitely many solutions.

Example 2 **Solve the system of equations by graphing.**

$y = 3x - 2$

$y = 3x - 4$

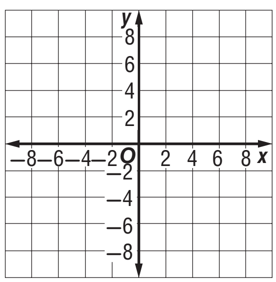


The graphs appear to be parallel lines. Because there is no coordinate pair that is a solution to both equations, there is no solution to this system of equations.

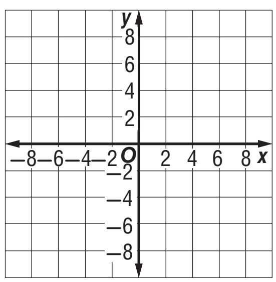
Exercises

Solve each system of equations by graphing.

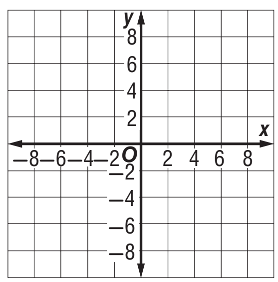
1. $y = 2x$
$y = x + 3$



2. $y = -3x$
$y = -2x - 2$



3. $y = \frac{1}{4}x + 2$
$y = \frac{1}{4}x - 3$



8-10 Study Guide and Intervention *(continued)*

Systems of Equations

Solve Systems by Substitution Systems of equations can also be solved algebraically by **substitution**.

Example **Solve the system of equations by substitution.**

$y = x + 5$

$y = 8$

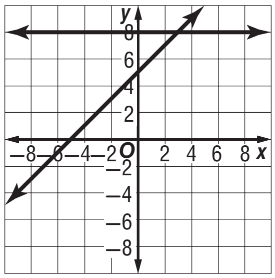
Replace y with 8 in the first equation.

$y = x + 5$ Write the first equation.

$8 = x + 5$ Replace y with 8.

$3 = x$ Solve for x.

The solution of this system of equations is (3, 8). You can check the solution by graphing. The graphs appear to intersect at (3, 8), so the solution is correct.



Exercises

Solve each system of equations by substitution.

1. $y = 6 + x$
$y = 1$

2. $y = 7 - x$
$y = 12$

3. $y = 3x$
$y = 21$

4. $y = 2x$
$y = -4$

5. $y = 2x - 6$
$y = -2$

6. $y = 4x + 11$
$y = 3$

7. $y = 6x - 21$
$y = -3$

8. $y = 3x + 14$
$y = 2$

9. $y = -2x - 8$
$y = 6$

10. $x + y = 17$
$y = 5$

11. $y + 2x = 12$
$y = x$

12. $3y - 2x = 20$
$y = 2x$

13. $5x - 2y = 22$
$y = 3x$

14. $6x - 3y = 27$
$y = -x$

15. $-y + 6x = 30$
$y = 4x$

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9-1 Study Guide and Intervention

Powers and Exponents

Use Exponents A number that is expressed using an exponent is called a **power.** The **base** is the number that is multiplied. The **exponent** tells how many times the base is used as a factor. So, 4^3 has a base of 4 and an exponent of 3, and $4^3 = 4 \cdot 4 \cdot 4 = 64$.

base → 4^3 ← exponent

power

Any number, except 0, raised to the zero power is defined to be 1.

$1^0 = 1$ $2^0 = 1$ $3^0 = 1$ $4^0 = 1$ $5^0 = 1$ $x^0 = 1, x \neq 0$

Example **Write each expression using exponents.**

a. $\mathbf{10 \cdot 10 \cdot 10 \cdot 10 \cdot 10}$

The base is 10. It is a factor 5 times, so the exponent is 5.

$10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 = 10^5$

b. $\mathbf{(-9) \cdot (-9) \cdot (-9) \cdot (-9) \cdot (-9) \cdot (-9)}$

The base is 9. It is a factor 6 times, so the exponent is 6.

$(-9) \cdot (-9) \cdot (-9) \cdot (-9) \cdot (-9) \cdot (-9) = (-9)^6$

c. $\mathbf{(p + 2)(p + 2)(p + 2)}$

The base is $p + 2$. It is a factor 3 times, so the exponent is 3.

$(p + 2)(p + 2)(p + 2) = (p + 2)^3$

Exercises

Write each expression using exponents.

1. $5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 \cdot 5$

2. $(-7)(-7)(-7)$

3. $4 \cdot 4$

4. $8 \cdot 8 \cdot 8$

5. $(-2) \cdot (-2) \cdot (-2) \cdot (-2)$

6. $\left(\frac{1}{3}\right) \cdot \left(\frac{1}{3}\right) \cdot \left(\frac{1}{3}\right) \cdot \left(\frac{1}{3}\right) \cdot \left(\frac{1}{3}\right)$

7. $(0.4) \cdot (0.4) \cdot (0.4)$

8. $d \cdot d \cdot d \cdot d$

9. $m \cdot m \cdot m \cdot m \cdot m \cdot m \cdot m$

10. $x \cdot x \cdot y \cdot y$

11. $(z - 4)(z - 4)$

12. $3(-t)(-t)(-t)$

9-1 Study Guide and Intervention *(continued)*

Powers and Exponents

Evaluate Expressions When evaluating expressions with exponents you must follow the order of operations.

Order of Operations

1. Simplify expressions inside grouping symbols.
2. Evaluate all powers.
3. Multiply and divide in order from left to right.
4. Add and subtract in order from left to right.

Example 1 **ART An artist is painting a mural that will look like a quilt square. The mural will have an area of 8^2 square feet. How many square feet is this?**

$8^2 = 8 \cdot 8$ — 8 is a factor 2 times.

$= 64$ — Simplify.

The area of the mural will be 64 square feet.

Example 2 **Evaluate $x^2 - 4$ if $x = -6$.**

$x^2 - 4 = (-6)^2 - 4$ — Replace x with -6.

$= (-6)(-6) - 4$ — -6 is a factor 2 times.

$= 36 - 4$ — Multiply.

$= 32$ — Subtract.

Exercises

Evaluate each expression.

1. 7^3 **2.** 3^6 **3.** $(-6)^3$ **4.** $\left(\frac{1}{5}\right)^4$

5. $(-4)^5$ **6.** 2^8 **7.** $3^3 \cdot 6$ **8.** $8^3 \cdot 9$

9. $7^2 \cdot 5$ **10.** $4^2 \cdot 5^2$ **11.** $(-3)^2 \cdot (-2)^3$ **12.** $8^2 \cdot 6^3$

Evaluate each expression if $g = 3$, $h = -1$, and $m = 9$.

13. g^5 **14.** $5g^2$ **15.** $g^2 - m$

16. hm^2 **17.** $g^3 + 2h$ **18.** $m + hg^3$

19. $4(2m - 3)^2$ **20.** $-2(g^3 + 1)$ **21.** $5(h^4 - m^2)$

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9-2 Study Guide and Intervention

Prime Factorization

Write Prime Factorizations A **prime number** is a whole number that has exactly two unique factors, 1 and itself. A **composite number** is a whole number that has more than two factors. Zero and 1 are neither prime nor composite.

Example 1 **Determine whether each number is *prime* or *composite*.**

a. 29

The only factors of 29 are 1 and 29, so 29 is a prime number.

b. 39

Find the factors of 39 by listing whole number pairs whose product is 39.

$39 \times 1 = 39$ $\qquad$ $13 \times 3 = 39$

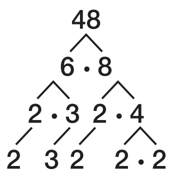
The factors of 39 are 1, 3, 13, and 39. Since the number has more than two factors, it is a composite number.

Any composite number can be written as a product of prime numbers. A factor tree can be used to find the prime factorization.

To make a factor tree:

1. Write the number that you are factoring at the top.
2. Choose any pair of whole number factors of the number.
3. Continue to factor any number that is not prime.

Example 2 **Find the prime factorization of 48.**



48 is the number to be factored.

Find any pair of whole number factors of 48.

Continue to factor any number that is not prime.

The factor tree is complete when there is a row of prime numbers.

The prime factorization of 48 is $2 \cdot 2 \cdot 2 \cdot 2 \cdot 3$ or $2^4 \cdot 3$.

Exercises

Determine whether each number is *prime* or *composite*.

1. 27 **2.** 151 **3.** 77 **4.** 25

5. 92 **6.** 49 **7.** 101 **8.** 81

Write the prime factorization of each number. Use exponents for repeated factors.

9. 16 **10.** 45 **11.** 78 **12.** 70

13. 50 **14.** 102 **15.** 76 **16.** 56

9-2 Study Guide and Intervention *(continued)*

Prime Factorization

Factor Monomials **Monomials** are numbers, variables, or products of numbers and/or variables. Examples of monomials and non-monomials are given below.

Monomials	Not Monomials
$38m$, 4, r	$38m + 5$, $4 - x$, $r^2 - s^2$

In algebra, monomials can be factored as a product of prime numbers and variables with no exponent greater than 1. So, $8x^2$ factors as $2 \cdot 2 \cdot 2 \cdot x \cdot x$. Negative coefficients can be factored using -1 as a factor.

Example **Factor each monomial.**

a. $3g^3h^2$

$3g^3h^2 = 3 \cdot g \cdot g \cdot g \cdot h \cdot h$ $g^3 = g \cdot g \cdot g$; $h^2 = h \cdot h$

b. $-12b^3c^4$

$-12b^3c^4 = -1 \cdot 2 \cdot 2 \cdot 3 \cdot b^3 \cdot c^4$ $-12 = -1 \cdot 2 \cdot 2 \cdot 3$

$= -1 \cdot 2 \cdot 2 \cdot 3 \cdot b \cdot b \cdot b \cdot c \cdot c \cdot c \cdot c$ $b^3 \cdot c^4 = b \cdot b \cdot b \cdot c \cdot c \cdot c \cdot c$

Exercises

Factor each monomial.

1. $21t$

2. $36xy$

3. $-45c^2$

4. $13b^4$

5. $6m^3$

6. $-20xy^2$

7. $a^2b^2c^3$

8. $25h$

9. $-6f^3g^3$

10. $100k^2l$

11. $-80s^4t^2$

12. $46p^3q^5$

13. $t^2u^3v^4$

14. $24ab^2c^4$

15. $-35x^3y^3$

16. $16r^2s^2t^2$

9-3 Study Guide and Intervention

Multiplying and Dividing Monomials

Multiply Monomials When multiplying powers with the same base, add the exponents.

Symbols	$a^m \cdot a^n = a^{m+n}$
Example	$4^2 \cdot 4^5 = 4^{2+5}$ or 4^7

Example 1 **Find each product.**

a. $\mathbf{5^7 \cdot 5}$

$5^7 \cdot 5 = 5^7 \cdot 5^1$	$5 = 5^1$
$= 5^{7+1}$	Product of Powers Property; the common base is 5.
$= 5^8$	Add the exponents.

b. $\mathbf{7^3 \cdot 7^2}$

$7^3 \cdot 7^2 = 7^{3+2}$	Product of Powers Property; the common base is 7.
$= 7^5$	Add the exponents.

Example 2 **Find each product.**

a. $\mathbf{g^3 \cdot g^6}$

$g^3 \cdot g^6 = g^{3+6}$	Product of Powers Property; the common base is *g*.
$= g^9$	Add the exponents.

b. $\mathbf{2a^2 \cdot 3a}$

$2a^2 \cdot 3a = 2 \cdot 3 \cdot a^2 \cdot a$	Commutative Property of Multiplication
$= 2 \cdot 3 \cdot a^{2+1}$	Product of Powers Property; the common base is *a*.
$= 2 \cdot 3 \cdot a^3$	Add the exponents.
$= 6a^3$	Multiply.

Exercises

Find each product. Express using exponents.

1. $4^7 \cdot 4^6$

2. $v^5 \cdot v^4$

3. $(f^3)(f^9)$

4. $(-31^4)(-31^2)$

5. $(-cr^5)(-r^2)$

6. $22^5 \cdot 22^5$

7. $7h(5h^3)$

8. $-10x^2(7x^3)$

9. $5p^3 \cdot (-4p)$

10. $3d^3 \cdot 12d^3$

11. $(-14x) \cdot x$

12. $9z^3 \cdot 2z \cdot (-z^4)$

13. $3^8 \cdot 3^3$

14. $-7u^6(-6u^5)$

15. $-5m^3(4m^6)$

9-3 Study Guide and Intervention *(continued)*

Multiplying and Dividing Monomials

Divide Monomials When dividing powers with the same base, subtract the exponents.

Symbols	$\frac{a^m}{a^n} = a^{m-n}$, where $a \neq 0$
Example	$\frac{5^6}{5^2} = 5^{6-2}$ or 5^4

Example 1 **Find each quotient.**

a. $\frac{(-8)^4}{(-8)^2}$

$\frac{(-8)^4}{(-8)^2} = (-8)^{4-2}$ Quotient of Powers Property; the common base is (–8).

$= (-8)^2$ Subtract the exponents.

b. $\frac{a^7}{a^3}$

$\frac{a^7}{a^3} = a^{7-3}$ Quotient of Powers Property; the common base is *a*.

$= a^4$ Subtract the exponents.

Example 2 **RIVERS The Mississippi River is approximately 3^7 miles long. The Kentucky River is approximately 3^5 miles long. About how many times as long is the Mississippi River than the Kentucky River?**

Write a division expression to compare the lengths.

$\frac{3^7}{3^5} = 3^{7-5}$ Quotient of Powers Property

$= 3^2$ or 9 Subtract the exponents. Simplify.

So, the Mississippi River is approximately 9 times as long as the Kentucky River.

Exercises

Find each quotient. Express using exponents.

1. $\frac{7^5}{7^2}$ **2.** $\frac{1^8}{1^6}$ **3.** $\frac{(-12)^3}{(-12)^3}$ **4.** $\frac{c^{20}}{c^{13}}$

5. $\frac{(-p^{18})}{(-p^{12})}$ **6.** $\frac{2w^3}{2w}$ **7.** $\frac{e^{10}}{e^3}$ **8.** $\frac{k^9}{k}$

9. $3v^3 \div 3v$ **10.** $12x^6 \div 12x^2$ **11.** $(-2a^5) \div (-2a)$ **12.** $5j^8 \div 5j^3$

9-4 Study Guide and Intervention

Negative Exponents

Negative Exponents Extending the pattern below shows that $4^{-1} = \frac{1}{4}$ or $\frac{1}{4^1}$.

$4^2 = 16$
÷ 4
$4^1 = 4$
÷ 4
$4^0 = 1$
÷ 4
$4^{-1} = \frac{1}{4}$

This suggests the following definition.

$a^{-n} = \frac{1}{a^n}$ for $a \neq 0$ and any whole number n. Example: $6^{-4} = \frac{1}{6^4}$

For $a \neq 0$, $a^0 = 1$. Example: $9^0 = 1$

Example 1 **Write each expression using a positive exponent.**

a. 3^{-4}

$3^{-4} = \frac{1}{3^4}$ Definition of negative exponent

b. y^{-2}

$y^{-2} = \frac{1}{y^2}$ Definition of negative exponent

Example 2 **Write each fraction as an expression using a negative exponent other than −1.**

a. $\frac{1}{6^3}$

$\frac{1}{6^3} = 6^{-3}$ Definition of negative exponent

b. $\frac{1}{81}$

$\frac{1}{81} = \frac{1}{9^2}$ Definition of exponent

$= 9^{-2}$ Definition of negative exponent

Exercises

Write each expression using a positive exponent.

1. 6^{-4} **2.** $(-7)^{-8}$ **3.** b^{-6} **4.** n^{-1}

5. $(-2)^{-5}$ **6.** 10^{-3} **7.** j^{-9} **8.** a^{-2}

Write each fraction as an expression using a negative exponent other than −1.

9. $\frac{1}{2^2}$ **10.** $\frac{1}{13^4}$ **11.** $\frac{1}{25}$ **12.** $\frac{1}{49}$

13. $\frac{1}{3^3}$ **14.** $\frac{1}{9^2}$ **15.** $\frac{1}{121}$ **16.** $\frac{1}{27}$

9-4 Study Guide and Intervention *(continued)*

Negative Exponents

Evaluate Expressions Algebraic expressions with negative exponents can be written using positive exponents and then evaluated.

Example 1 **Evaluate b^{-2} if $b = 3$.**

$b^{-2} = 3^{-2}$	Replace *b* with 3.
$= \frac{1}{3^2}$	Definition of negative exponent
$= \frac{1}{9}$	Find 3^2.

Example 2 **Evaluate $8c^{-4}$ if $c = 2$.**

$8c^{-4} = 8(2)^{-4}$	Replace *c* with 2.
$= 8 \cdot \frac{1}{2^4}$	Definition of negative exponent
$= 8 \cdot \frac{1}{16}$	Find 2^4.
$= \overset{1}{\cancel{8}} \cdot \frac{1}{\underset{2}{\cancel{16}}}$	Simplify.
$= \frac{1}{2}$	Simplify.

Exercises

Evaluate each expression if $m = -4$, $n = 1$, and $p = 6$.

1. p^{-2} **2.** m^{-3} **3.** $(np)^{-1}$ **4.** 3^m

5. p^m **6.** $(2m)^{-2}$ **7.** m^{-p} **8.** $(mp)^{-n}$

9. 4^m **10.** -3^{-n} **11.** mp^{-2} **12.** pm^{-2}

9-5 Study Guide and Intervention

Scientific Notation

Scientific Notation Numbers like 5,000,000 and 0.0005 are in **standard form** because they do not contain exponents. A number is expressed in **scientific notation** when it is written as a product of a factor and a power of 10. The factor must be greater than or equal to 1 and less than 10.

By definition, a number in scientific notation is written as $a \times 10^n$, where $1 \le a < 10$ and n is an integer.

Example 1 **Express each number in standard form.**

a. 6.32×10^5

$6.32 \times 10^5 = 6.32 \times 100{,}000$ — $10^5 = 100{,}000$

$= 632{,}000$ — Move the decimal point 5 places to the right.

b. 7.8×10^{-6}

$7.8 \times 10^{-6} = 7.8 \times 0.000001$ — $10^{-6} = 0.000001$

$= 0.0000078$ — Move the decimal point 6 places to the left.

Example 2 **Express each number in scientific notation.**

a. 62,000,000

To write in scientific notation, place the decimal point after the first nonzero digit, then find the power of 10.

$62{,}000{,}000 = 6.2 \times 10{,}000{,}000$ — The decimal point moves 7 places.

$= 6.2 \times 10^7$ — The exponent is positive.

b. 0.00025

$0.00025 = 2.5 \times 0.0001$ — The decimal point moves 4 places.

$= 2.5 \times 10^{-4}$ — The exponent is negative.

Exercises

Express each number in standard form.

1. 4.12×10^6

2. 5.8×10^2

3. 9.01×10^{-3}

4. 6.72×10^{-7}

5. 8.72×10^4

6. 4.44×10^{-5}

7. 1.034×10^9

8. 3.48×10^{-4}

9. 6.02×10^{-6}

Express each number in scientific notation.

10. 12,000,000,000

11. 5000

12. 0.00475

13. 0.00007463

14. 235,000

15. 0.000377

16. 7,989,000,000

17. 0.0000403

18. 13,000,000

9-5 Study Guide and Intervention *(continued)*

Scientific Notation

Compare and Order Numbers You can compare and order numbers in scientific notation without converting them into standard form.

> To compare numbers in Scientific Notation, compare the exponents.
> - If the exponents are positive, the number with the greatest exponent is the greatest.
> - If the exponents are negative, the number with the least exponent is the least.
> - If the exponents are the same, compare the factors.

Example 1 **Compare each set of numbers using <, > or =.**

a. 2.097×10^5 ● 3.12×10^3 Compare the exponents: $5 > 3$.
So, $2.097 \times 10^5 > 3.12 \times 10^3$.

b. 8.706×10^{-5} ● 8.809×10^{-5} The exponents are the same, so compare the factors: $8.706 < 8.809$.
So, $8.706 \times 10^{-5} < 8.809 \times 10^{-5}$.

Example 2 **ATOMS The table shows the weight of protons, neutrons, and electrons. Rank the particles in order from heaviest to lightest.**

Particle	Weight
Electron	9.109×10^{-31}
Proton	1.672×10^{-27}
Neutron	1.674×10^{-27}

Step 1: Order the numbers according to their exponents. The electron has an exponent of −31. So, it has the least weight.

Step 2: Order the numbers with the same exponent by comparing the factors.

$$1.672 < 1.674$$

So, $1.674 \times 10^{-27} > 1.672 \times 10^{-27} > 9.109 \times 10^{-31}$.
The order from heaviest to lightest is neutron, proton, and electron.

Exercises

Choose the greater number in each pair.

1. 4.9×10^4, 9.9×10^{-4}

2. 2.004×10^3, 2.005×10^{-2}

3. 3.2×10^2, 700

4. 0.002, 3.6×10^{-4}

Order each set of numbers from least to greatest.

5. 6.9×10^3, 7.6×10^{-6}, 7.1×10^3, 6.8×10^4

6. 4.02×10^{-8}, 4.15×10^{-3}, 4.2×10^2, 4.0×10^{-8}

7. 8.16×10^6, 81,600,000, 8.06×10^6, 8.2×10^{-6}

8. 210,000,000, 2.05×10^8, 21,500,000, 2.15×10^6

9-6 Study Guide and Intervention

Powers of Monomials

Power of a Power You can use the property for finding the *product* of powers to find a property for finding the *power* of a power.

$(h^3)^4 = (h^3)(h^3)(h^3)(h^3)$ — The meaning of $(h^3)^4$ is (h^3) should be used as a factor 4 times.

$= h^{3+3+3+3}$ — Product of Powers Property

$= h^{12}$

The result of multiplying h^3 by itself 4 times was the same as multiplying the two exponents.

Power of a Power Property

To find the power of a power, multiply the exponents.

$(a^m)^n = a^{m \cdot n}$

Example **Simplify.**

a. $(4^3)^6$

$(4^3)^6 = 4^{3 \cdot 6}$ — Power of a Power

$= 4^{18}$ — Simplify.

b. $(c^2)^7$

$(c^2)^7 = c^{2 \cdot 7}$ — Power of a Power

$= c^{14}$ — Simplify.

Exercises

Simplify.

1. $(7^3)^4$ **2.** $(12^7)^3$ **3.** $(8^5)^7$ **4.** $(22^3)^2$

5. $(x^8)^5$ **6.** $(y^2)^8$ **7.** $(b^3)^3$ **8.** $(r^6)^4$

9. $(4^3)^{-5}$ **10.** $(-6^6)^2$ **11.** $(5^3)^{-6}$ **12.** $(-10^{10})^{-3}$

13. $(t^4)^{-2}$ **14.** $(-s^4)^9$ **15.** $(e^3)^{-6}$ **16.** $(d^6)^7$

9-6 Study Guide and Intervention *(continued)*

Powers of Monomials

Power of a Product

The Power of a Power Property can be extended to find the power of a product.

$(3d^2)^3 = (3d^2)(3d^2)(3d^2)$ The meaning of $(3d^2)^3$ is multiplying $(3d^2)$ by itself 3 times.

$= 3^3 \cdot (d^2)^3$

$= 3^3 \cdot (d^2) \cdot (d^2) \cdot (d^2)$ The meaning of $(d^2)^3$ is multiplying (d^2) by itself 3 times.

$= 3^3 \cdot d^{2+2+2}$ Product of Powers Property

$= 27 \cdot d^6$ or $27d^6$

Power of a Product Property

To find the power of a product, find the power of each factor and multiply.

$(ab)^m = a^m b^m$, for all numbers a and b and any integer m

Example **Simplify.**

a. $(7x^4)^2$

$(7x^4)^2 = 7^2 \cdot (x^4)^2$ Power of a Product

$= 7^2 \cdot x^{4 \cdot 2}$ Power of a Power

$= 49 \cdot x^8$ Simplify.

b. $(3a^4b^6)^2$

$(3a^4b^6)^2 = 3^2 \cdot (a^4)^2 \cdot (b^6)^2$ Power of a Product

$= 3^2 \cdot (a^{4 \cdot 2}) \cdot (b^{6 \cdot 2})$ Power of a Power

$= 9a^8b^{12}$ Simplify.

Exercises

Simplify.

1. $(6x^5)^3$ **2.** $(5b^{-3})^4$ **3.** $(12h^7)^2$ **4.** $(-8j^2)^3$

5. $(11z^{-9})^2$ **6.** $(7a^6)^3$ **7.** $(4g^{-2})^4$ **8.** $(2k^3)^5$

9. $(6p^7q^6)^3$ **10.** $(-9m^9n^4)^3$ **11.** $(10f^{-2}g^9)^5$ **12.** $(5d^7e^{10})^3$

13. $(-4s^6t^8)^4$ **14.** $(3r^5s^3)^4$ **15.** $(8a^2b^3)^3$ **16.** $(-10v^{-5}w^3)^4$

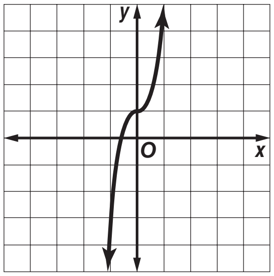
9-7 Study Guide and Intervention

Linear and Nonlinear Functions

Graphs of Nonlinear Functions Linear functions are relations with a constant rate of change. Graphs of linear functions are straight lines. **Nonlinear functions** do not have a constant rate of change. Graphs of nonlinear functions are not straight lines.

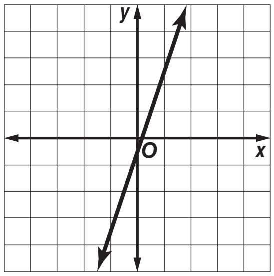
Example **Determine whether each graph represents a *linear* or *nonlinear* function. Explain.**

a.



This graph is a curve, not a straight line. So, it represents a nonlinear function.

b.

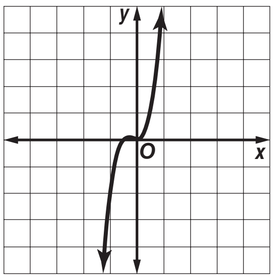


This graph is a line. So, it represents a linear function.

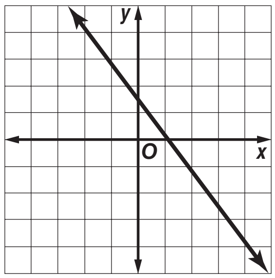
Exercises

Determine whether each graph represents a *linear* or *nonlinear* function. Explain.

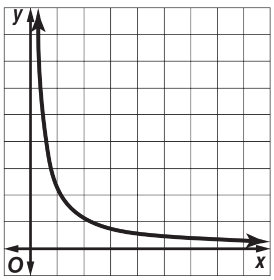
1.



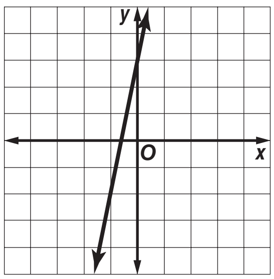
2.



3.



4.



9-7 Study Guide and Intervention *(continued)*

Linear and Nonlinear Functions

Equations and Tables Linear functions have constant rates of change. Their graphs are straight lines and their equations can be written in the form $y = mx + b$. Nonlinear functions do not have constant rates of change and their graphs are not straight lines.

Example 1 **Determine whether each equation represents a *linear* or *nonlinear* function. Explain.**

a. $y = 9$

This is linear because it can be written as $y = 0x + 9$.

b. $y = x^2 + 4$

This is nonlinear because the exponent of x is not 1, so the equation cannot be written in the form $y = mx + b$.

Tables can represent functions. A nonlinear function does not increase or decrease at a constant rate.

Example 2 **Determine whether each table represents a *linear* or *nonlinear* function. Explain.**

a.

x	y
0	–7
2	1
4	9
6	17

(x: + 2, + 2, + 2; y: + 8, + 8, + 8)

As x increases by 2, y increases by 8. The rate of change is constant, so this is a linear function.

b.

x	y
0	100
5	75
10	0
15	–125

(x: + 5, + 5, + 5; y: – 25, – 75, – 125)

As x increases by 5, y decreases by a greater amount each time. The rate of change is not constant, so this is a nonlinear function.

Exercises

Determine whether each equation or table represents a *linear* or *nonlinear* function. Explain.

1. $x + 3y = 9$

2. $y = \frac{8}{x}$

3. $y = 6x(x + 1)$

4. $y = 9 - 5x$

5.

x	y
0	24
2	14
4	4
6	–6

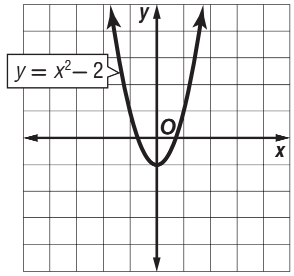
6.

x	y
1	1
2	8
3	27
4	64

9-8 Study Guide and Intervention

Quadratic Functions

Graph Quadratic Functions Functions which can be described by an equation of the form $y = ax^2 + bx + c$, where $a \neq 0$, are called **quadratic functions**. The graph of a quadratic equation takes the form shown to the right, which is called a **parabola**.

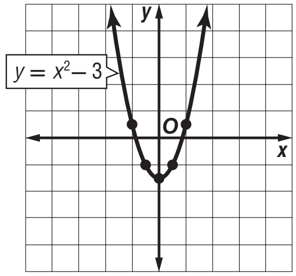


Just as with linear functions, you can graph quadratic functions by making a table of values.

Example **Graph $y = x^2 - 3$.**

Make a table of values, plot the ordered pairs, and connect the points with a curve.

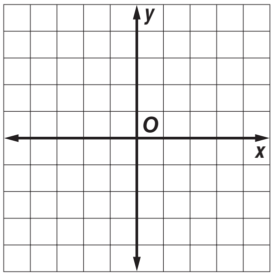
x	$y = x^2 - 3$	(x, y)
−2	$y = (-2)^2 - 3 = 1$	(−2, 1)
−1	$y = (-1)^2 - 3 = -2$	(−1, −2)
0	$y = (0)^2 - 3 = -3$	(0, −3)
1	$y = (1)^2 - 3 = -2$	(1, −2)
2	$y = (2)^2 - 3 = 1$	(2, 1)



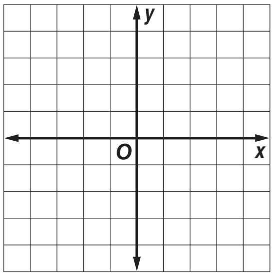
Exercises

Graph each function.

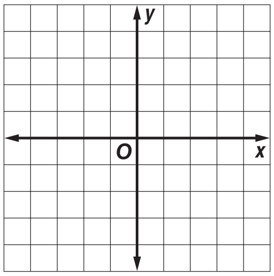
1. $y = x^2 + 2$



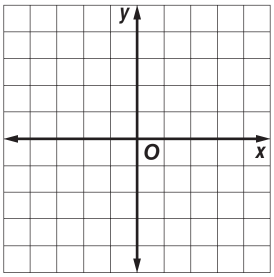
2. $y = -x^2 + 2$



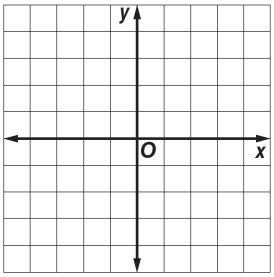
3. $y = x^2 - 2$



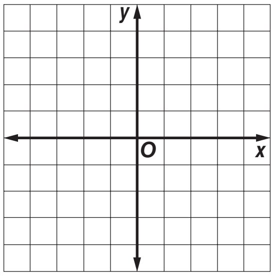
4. $y = 3x^2 - 1$



5. $y = \frac{1}{4}x^2$



6. $y = -2x^2 + 3$



9-8 Study Guide and Intervention *(continued)*

Quadratic Functions

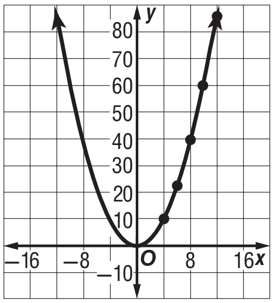
Use Quadratic Functions Many quadratic functions model real-world situations. You can use graphs of quadratic equations to analyze such situations.

Example **MAPS The principal of Smithville Elementary wants to paint a map of the U.S. on the cafeteria wall. Before the map can be painted, the rectangular space where the map will go must be painted white. The height of the rectangle will be $\frac{3}{5}$ the width.**

a. Graph the equation that gives the area for the rectangle for different lengths and widths. What is the area of the rectangle with a width of 10 feet? What is the length?

x	$y = \frac{3}{5}x^2$	(x, y)
4	$y = \frac{3}{5}(4)^2$	(4, 9.6)
6	$y = \frac{3}{5}(6)^2$	(6, 21.6)
8	$y = \frac{3}{5}(8)^2$	(8, 38.4)
10	$y = \frac{3}{5}(10)^2$	(10, 60)
12	$y = \frac{3}{5}(12)^2$	(12, 86.4)

Since area = length × width, use the quadratic equation $y = \frac{3}{5}x^2$, where y = the area and x = the width.



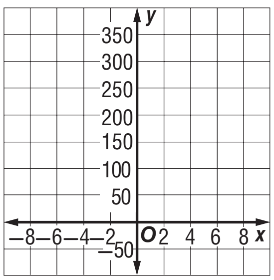
The area of the rectangle when the width is 10 feet is 60 square feet. The length is 6 feet.

b. What values of the domain and range are unreasonable? Explain.

Unreasonable values for the domain and range would be any negative numbers because neither the length nor the width can be negative.

Exercise

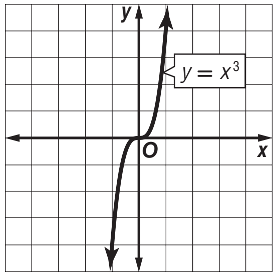
1. **GRAVITY** An object is dropped from a height of 300 feet. The equation that gives the object's height in feet h as a function of time t is $h = -16t^2 + 300$. Graph this equation and interpret your graph. What was the height of the object after 4 seconds?



9-9 Study Guide and Intervention

Cubic and Exponential Functions

Cubic Functions Functions which can be described by an equation of the form $y = ax^3 + bx^2 + cx + d$, where $a \neq 0$, are called **cubic functions**. The graph of a cubic equation takes the form shown to the right.

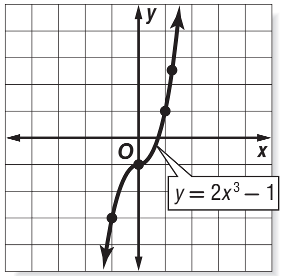


Just as with linear and quadratic functions, you can graph cubic functions by making a table of values.

Example **Graph $y = 2x^3 - 1$.**

Make a table of values, plot the ordered pairs, and connect the points with a curve.

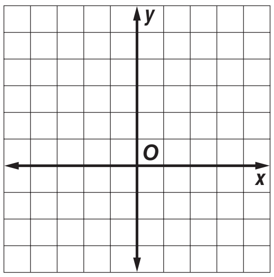
x	$y = 2x^3 - 1$	(x, y)
−1	$y = 2(-1)^3 - 1 = -3$	(−1, −3)
0	$y = 2(0)^3 - 1 = -1$	(0, −1)
1	$y = 2(1)^3 - 1 = 1$	(1, 1)
1.2	$y = 2(1.2)^3 - 1 \approx 2.5$	(1.2, 2.5)



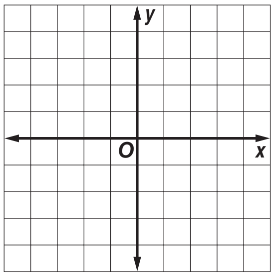
Exercises

Graph each function.

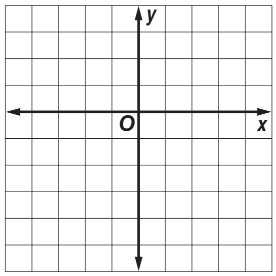
1. $y = x^3 + 2$



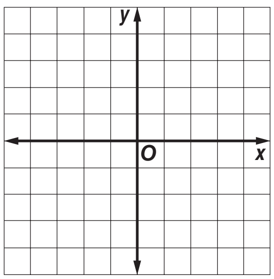
2. $y = -x^3 + 2$



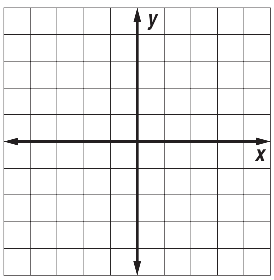
3. $y = x^3 - 2$



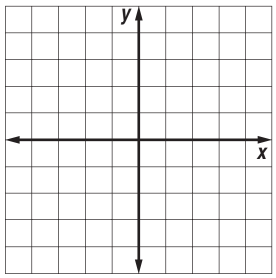
4. $y = 2x^3$



5. $y = -2x^3 + 2$



6. $y = \frac{5}{6}x^3 - 1$



9-9 Study Guide and Intervention *(continued)*

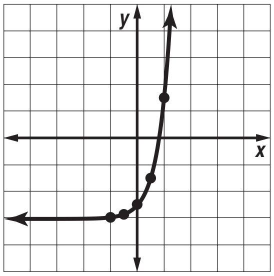
Cubic and Exponential Functions

Exponential Functions In linear, quadratic, and cubic functions, the variable is the base. Exponential functions are functions in which the variable is the exponent rather than the base. An **exponential function** is a function that can be described by an equation of the form $y = a^x + c$, where $a \neq 0$ and $a \neq 1$.

Example **Graph $y = 3^x - 6$.**

First, make a table of ordered pairs. Then graph the ordered pairs.

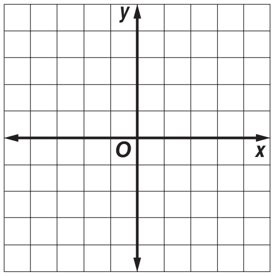
x	$y = 3^x - 6$	(x, y)
−2	$y = 3^{-2} - 6 \approx -5.9$	(−2, −5.9)
−1	$y = 3^{-1} - 6 \approx -5.7$	(−1, −5.7)
0	$y = 3^0 - 6 = -5$	(0, −5)
1	$y = 3^1 - 6 = -3$	(1, −3)
2	$y = 3^2 - 6$	(2, 3)



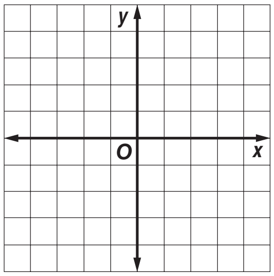
Exercises

Graph each function.

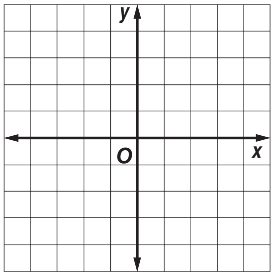
1. $y = 2^x$



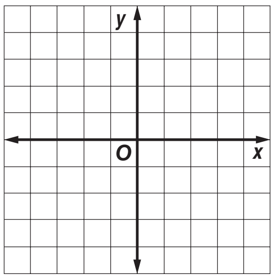
2. $y = 3^x + 2$



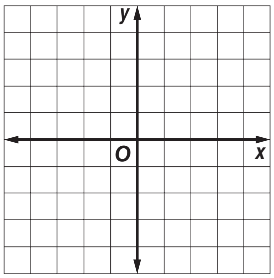
3. $y = 2^x - 1$



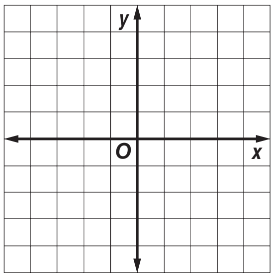
4. $y = 2^x + 1$



5. $y = 3^x - 3$



6. $y = 4^x - 6$



10-1 Study Guide and Intervention

Squares and Square Roots

Squares and Square Roots

- A **perfect square** is the square of an integer.
- A **square root** of a number is one of two equal factors of the number.
- A **radical sign**, $\sqrt{}$, is used to indicate a positive square root.
- Every positive number has a positive square root and a negative square root.
- The square root of a negative number, such as –64, is not real because the square of a number cannot be negative.

Example **Find each square root.**

a. $\sqrt{144}$

$\sqrt{144} = 12$ Find the positive square root of 144; $12^2 = 144$.

b. $-\sqrt{121}$

$-\sqrt{121} = -11$ Find the negative square root of 121; $11^2 = 121$.

c. $\pm\sqrt{49}$

$\pm\sqrt{49} = \pm 7$ Find both square roots of 49; $7^2 = 49$.

d. $\sqrt{-100}$

$\sqrt{-100}$ There is no real square root because no number times itself is equal to –100.

Exercises

Find each square root.

1. $\sqrt{25}$ **2.** $\sqrt{-25}$ **3.** $\sqrt{169}$

4. $-\sqrt{196}$ **5.** $\pm\sqrt{16}$ **6.** $\sqrt{-4}$

7. $\sqrt{400}$ **8.** $-\sqrt{81}$ **9.** $\pm\sqrt{225}$

10. $\sqrt{-9}$ **11.** $\sqrt{256}$ **12.** $-\sqrt{289}$

13. $\pm\sqrt{361}$ **14.** $-\sqrt{484}$ **15.** $\sqrt{1521}$

10-1 Study Guide and Intervention *(continued)*

Squares and Square Roots

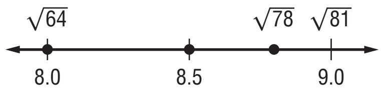
Estimate Square Roots When integers are not perfect squares, you can estimate square roots mentally by using perfect squares.

Example 1 **Estimate $\sqrt{78}$ to the nearest integer.**

$\sqrt{78}$

The first perfect square less than 78 is 64. $\sqrt{64} = 8$

The first perfect square greater than 78 is 81. $\sqrt{81} = 9$



The square root of 78 is between 8 and 9. Since 78 is closer to 81 than to 64, you can expect $\sqrt{78}$ to be closer to 9 than to 8.

If allowed, calculators can also be used to estimate square roots.

Example 2 **Use a calculator to find $\sqrt{34}$ to the nearest tenth.**

[2nd] [$\sqrt{\ }$] 34 [ENTER] 5.830951895 Use a calculator.

$\sqrt{34} \approx 5.8$ Round to the nearest tenth.

Exercises

Estimate each square root to the nearest integer. Do not use a calculator.

1. $\sqrt{11}$ **2.** $\sqrt{62}$ **3.** $\sqrt{29}$ **4.** $\sqrt{14}$

5. $\sqrt{96}$ **6.** $\sqrt{5}$ **7.** $\sqrt{41}$ **8.** $\sqrt{150}$

9. $\sqrt{53}$ **10.** $\sqrt{116}$ **11.** $\sqrt{84}$ **12.** $\sqrt{180}$

Use a calculator to find each square root to the nearest tenth.

13. $\sqrt{8}$ **14.** $\sqrt{115}$ **15.** $-\sqrt{21}$ **16.** $-\sqrt{88}$

17. $\sqrt{200}$ **18.** $\sqrt{42}$ **19.** $-\sqrt{67}$ **20.** $-\sqrt{136}$

21. $\sqrt{12}$ **22.** $\sqrt{50}$ **23.** $-\sqrt{250}$ **24.** $-\sqrt{86}$

NAME ______________________ DATE ____________ PERIOD ____________

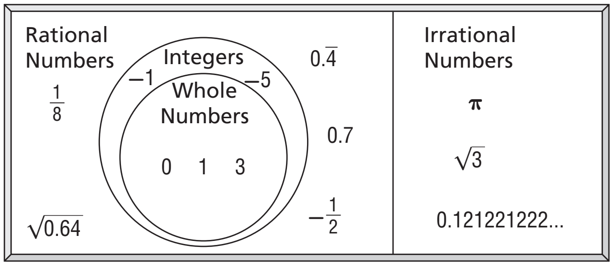
10-2 Study Guide and Intervention

The Real Number System

Identify and Compare Real Numbers

The set of **real numbers** consists of all whole numbers, integers, rational numbers, and irrational numbers.

- Rational numbers can be written as fractions.
- **Irrational numbers** are decimals that do not repeat or terminate.



Example 1 **Name all of the sets of numbers to which each real number belongs.**

a. 7
This number is a whole number, an integer, and a rational number.

b. $\mathbf{0.\overline{6}}$
This repeating decimal is a rational number because it is equivalent to $\frac{2}{3}$.

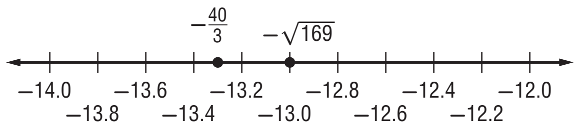
c. $\mathbf{\sqrt{71}}$
It is not the square root of a perfect square so it is irrational.

Example 2 **Replace ● with <, >, or = to make** $-\sqrt{169}$ **●** $-\frac{40}{3}$ **a true statement.**

Express each number as a decimal. Then, compare the decimals.

$-\sqrt{169} = -13.0$

$-\frac{40}{3} = -13.33333...$



Since -13.0 is greater than $-13.333...$, $-\sqrt{169} > -\frac{40}{3}$.

Exercises

Name all of the sets of numbers to which each real number belongs. Let W = whole numbers, Z = integers, Q = rational numbers, and I = irrational numbers.

1. 21 **2.** $\frac{3}{7}$ **3.** $\frac{8}{12}$

4. −5 **5.** 17 **6.** 0

7. 0.257 **8.** 0.9 **9.** $\sqrt{5}$

Replace each ● with <, >, or = to make a true statement.

10. $8.\overline{3}$ ● $\sqrt{65}$ **11.** $-3\frac{1}{8}$ ● $-\sqrt{14}$ **12.** $\sqrt{125}$ ● $\frac{45}{11}$

13. $-35.\overline{7}$ ● $-35\frac{7}{9}$ **14.** $\sqrt{200}$ ● 14.2 **15.** $99.\overline{27}$ ● $99\frac{2}{3}$

10-2 Study Guide and Intervention *(continued)*

The Real Number System

Solve Equations When a variable in an equation is within a radical symbol, it is called a "radical equation". By definition the following holds true: If $x^2 = y$, then $x = \pm\sqrt{y}$. The relationship can be used to solve equations involving squares. When solving equations for real-world problems, most solutions will not make sense with a negative square root, so in these cases only use the positive, or *principal*, square root.

Example **Solve each equation. Round to the nearest tenth, if necessary.**

a. $\boldsymbol{b^2 = 121}$

$b^2 = 121$	Write the equation.
$b = \pm\sqrt{121}$	Definition of square root
$b = 11$ and -11	Check $11 \cdot 11 = 121$ and $(-11) \cdot (-11) = 121$

The solutions are 11 and –11.

b. $\boldsymbol{6n^2 = 180}$

$6n^2 = 180$	Write the equation.
$n^2 = 30$	Divide each side by 6.
$n = \pm\sqrt{30}$	Definition of square root
$n \approx 5.5$ and -5.5	Use a calculator.

The solutions are 5.5 and –5.5.

Exercises

Solve each equation. Round to the nearest tenth, if necessary.

1. $x^2 = 9$

2. $t^2 = 25$

3. $4h^2 = 144$

4. $16t^2 = 784$

5. $y^2 = 30$

6. $4s^2 = 576$

7. $3a^2 = 243$

8. $n^2 = 51$

9. $5m^2 = 605$

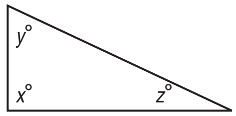
10. $r^2 = 10$

11. $7v^2 = 280$

12. $6u^2 = 504$

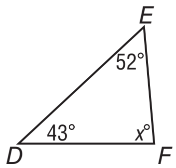
10-3 Study Guide and Intervention

Triangles

Angles of a Triangle		
Words	The sum of the measures of the angles of a triangle is 180°.	**Model**
Symbols	$x + y + z = 180$	

Example 1 **Find the value of x in $\triangle DEF$.**

$m\angle D + m\angle E + m\angle F = 180$	The sum of the measures is 180°.
$43 + 52 + x = 180$	$m\angle D = 43°$ and $m\angle E = 52°$
$95 + x = 180$	Simplify.
$x + 95 - 95 = 180 - 95$	Subtract 95 from each side.
$x = 85$	



Example 2 **The measures of the angles of $\triangle DEF$ are in the ratio 1:2:6. What are the measures of the angles?**

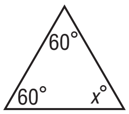
Let x represent the measure of the first angle, $2x$ the measure of a second angle, and $6x$ the measure of the third angle.

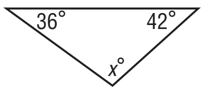
$x + 2x + 6x = 180$	Write the equation.
$9x = 180$	Combine like terms.
$\frac{9x}{9} = \frac{180}{9}$	Divide each side by 9.
$x = 20$	Simplify.

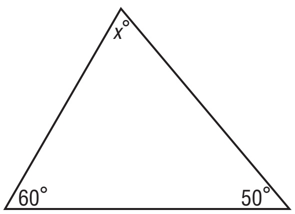
Since $x = 20$, $2x = 2(20)$ or 40, and $6x = 6(20)$, or 120. The measures of the angles are 20°, 40°, and 120°.

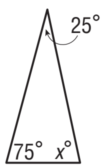
Exercises

Find the value of x in each triangle.

1. 

2. 

3. 

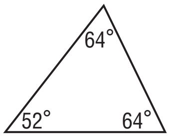
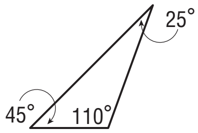
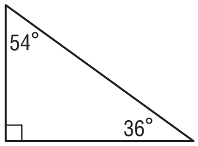
4. 

5. The measures of the angles of $\triangle XYZ$ are in the ratio 1:4:10. What are the measures of the angles?

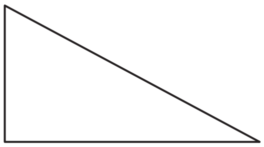
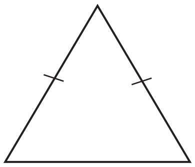
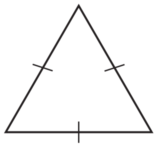
10-3 Study Guide and Intervention *(continued)*

Triangles

Classify Triangles Angles can be classified by their degree measure. **Acute angles** measure between 0° and 90°. An **obtuse angle** measures between 90° and 180°. A **right angle** measures 90°, and a **straight angle** measures 180°.

Classify Triangles by Angles		
Acute Triangle	**Obtuse Triangle**	**Right Triangle**
		
all acute angles	one obtuse angle	one right angle

Triangles can be classified by their sides. **Congruent** sides are sides that have the same length.

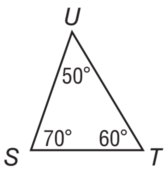
Classify Triangles by Sides		
Scalene Triangle	**Isosceles Triangle**	**Equilateral Triangle**
		
no congruent sides	at least two sides congruent	all sides congruent

Example **Classify the triangle by its angles and by its sides.**

$m\angle TUS < 90°$, $m\angle STU < 90°$, and $m\angle UST < 90°$, so ΔSTU has all acute angles.

ΔSTU has no two sides that are congruent.

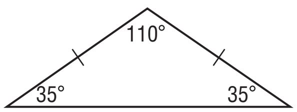
So, ΔSTU is an acute scalene triangle.

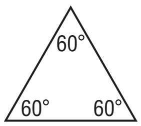


Exercises

Classify each triangle by its angles and by its sides.

1. 

2. 

3. 

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10-4 Study Guide and Intervention

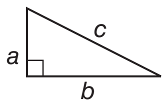
The Pythagorean Theorem

Use the Pythagorean Theorem In a right triangle, the sides adjacent to the right angle are called the **legs.** The side opposite the right angle is the **hypotenuse**. It is the longest side of a right triangle. The **Pythagorean Theorem** describes the relationship between the lengths of the legs and the hypotenuse for any right triangle.

Pythagorean Theorem

Words If a triangle is a right triangle, then the square of the length of the hypotenuse is equal to the sum of the squares of the lengths of the legs.

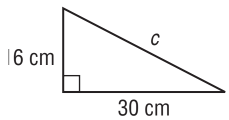
Model



Symbols $a^2 + b^2 = c^2$

Example **Find the length of the hypotenuse of the right triangle.**

$a^2 + b^2 = c^2$	Pythagorean Theorem
$16^2 + 30^2 = c^2$	Replace *a* with 16 and *b* with 30.
$256 + 900 = c^2$	Evaluate 16^2 and 30^2.
$1156 = c^2$	Add 256 and 900.
$\pm\sqrt{1156} = c$	Definition of square root
$34 = c$	Use the principal square root.

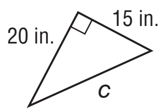


The length of the hypotenuse is 34 centimeters.

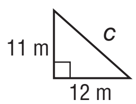
Exercises

Find the length of the hypotenuse of each right triangle. Round to the nearest tenth, if necessary.

1.



2.



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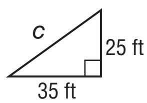

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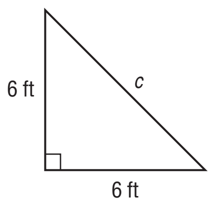

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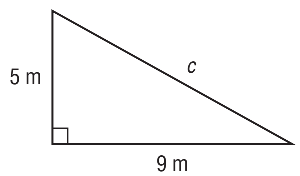

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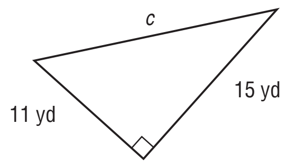


If *c* is the measure of the hypotenuse, find each missing measure. Round to the nearest tenth, if necessary.

7. $a = 18, b = 80, c = ?$

8. $a = ?, b = 70, c = 74$

9. $a = 14, b = ?, c = 22$

10. $a = ?, b = 48, c = 57$

10-4 Study Guide and Intervention *(continued)*

The Pythagorean Theorem

Use the Converse of the Pythagorean Theorem

The Pythagorean Theorem is written in if-then form.

If a triangle is a right triangle, **then** $c^2 = a^2 + b^2$.

If you reverse the statements after *if* and *then*, you form the **converse** of the Pythagorean Theorem.

If $c^2 = a^2 + b^2$, **then** a triangle is a right triangle.

Since the converse of the Pythagorean Theorem is true, you can use it to determine whether or not a triangle is a right triangle.

Example **The measures of three sides of a triangle are given. Determine whether each triangle is a right triangle.**

a. 6 ft, 7 ft, 10 ft

$a^2 + b^2 = c^2$	Pythagorean Theorem
$6^2 + 7^2 \stackrel{?}{=} 10^2$	$a = 6$, $b = 7$, $c = 10$
$36 + 49 \stackrel{?}{=} 100$	Evaluate.
$85 \neq 100$	Simplify.

The triangle is *not* a right triangle.

b. 7 m, 24 m, 25 m

$a^2 + b^2 = c^2$	Pythagorean Theorem
$7^2 + 24^2 \stackrel{?}{=} 25^2$	$a = 7$, $b = 24$, $c = 25$
$49 + 576 \stackrel{?}{=} 625$	Evaluate.
$625 = 625$	Simplify.

The triangle is a right triangle.

Exercises

The lengths of three sides of a triangle are given. Determine whether each triangle is a right triangle.

1. $a = 8$, $b = 15$, $c = 17$

2. $a = 5$, $b = 12$, $c = 13$

3. $a = 9$, $b = 38$, $c = 38$

4. $a = 13$, $b = 36$, $c = 40$

5. $a = 5$, $b = 9$, $c = 13$

6. $a = 15$, $b = 20$, $c = 25$

7. $a = 9$, $b = 13$, $c = 21$

8. $a = 18$, $b = 24$, $c = 30$

9. $a = 20$, $b = 24$, $c = 26$

10. $a = 16$, $b = 30$, $c = 34$

11. $a = 25$, $b = 31$, $c = 37$

12. $a = 21$, $b = 29$, $c = 42$

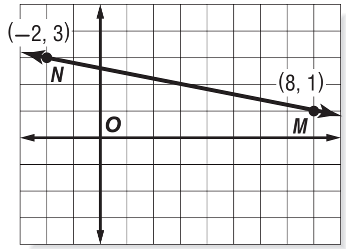
10-5 Study Guide and Intervention

The Distance Formula

Distance Formula On a coordinate plane, the distance d between two points with coordinates (x_1, y_1) and (x_2, y_2) is given by $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.

Example **Find the distance between $M(8, 1)$ and $N(-2, 3)$. Round to the nearest tenth, if necessary.**

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$	Distance Formula
$MN = \sqrt{(8 - (-2))^2 + (1 - 3)^2}$	$(x_1, y_1) = (-2, 3)$, $(x_2, y_2) = (8, 1)$
$MN = \sqrt{(10)^2 + (-2)^2}$	Simplify.
$MN = \sqrt{100 + 4}$	Evaluate 10^2 and $(-2)^2$.
$MN = \sqrt{104}$	Add 100 and 4.
$MN \approx 10.2$	Take the square root.



The distance between points M and N is about 10.2 units.

Exercises

Find the distance between each pair of points. Round to the nearest tenth, if necessary.

1. $A(3, 1)$, $B(2, 5)$

2. $C(-2, -4)$, $D(3, 7)$

3. $E(5, -3)$, $F(4, 2)$

4. $G(-6, 5)$, $H(-4, -3)$

5. $I(-4, -3)$, $J(4, 4)$

6. $K(5, 0)$, $L(-2, 1)$

7. $M(2, 1)$, $N(6, 5)$

8. $O(0, 0)$, $P(-5, 6)$

9. $Q(3, 5)$, $R(4, 2)$

10. $S(-6, -4)$, $T(-5, 6)$

11. $U(2, 1)$, $V(4, 4)$

12. $W(5, 1)$, $X(-2, -1)$

13. $Y(-5, -3)$, $Z(2, 5)$

14. $A(8, -1)$, $B(3, -1)$

15. $C(0, 0)$, $D(2, 4)$

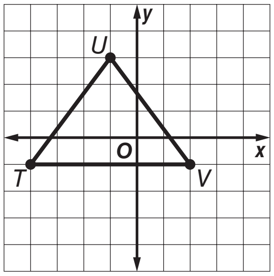
16. $E(-5, 3)$, $F(4, 7)$

10-5 Study Guide and Intervention *(continued)*

The Distance Formula

Apply the Distance Formula Knowing the coordinates of points on a figure allows you to draw conclusions about it and solve problems about the figure on the coordinate plane.

Example **GEOMETRY Classify ΔTUV by its sides. Then find its perimeter to the nearest tenth.**



Step 1 Use the Distance Formula to find the length of each side of the triangle.

Side $\overline{TU}$ has endpoints $T(-4, -1)$ and $U(-1, 3)$.

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$TU = \sqrt{[(-1) - (-4)]^2 + [3 - (-1)]^2}$$

$$TU = \sqrt{(3)^2 + (4)^2}$$

$$TU = \sqrt{9 + 16} \text{ or } \sqrt{25}$$

Side $\overline{UV}$ has endpoints $U(-1, 3)$ and $V(2,-1)$.

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$UV = \sqrt{[2 - (-1)]^2 + [(-1) - 3]^2}$$

$$UV = \sqrt{(3)^2 + (-4)^2}$$

$$UV = \sqrt{9 + 16} \text{ or } \sqrt{25}$$

Side $\overline{VT}$ has endpoints $V(2,-1)$ and $T(-4, -1)$.

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$VT = \sqrt{[(-4) - (-2)]^2 + [(-1) - (-1)]^2}$$

$$VT = \sqrt{(-6)^2 + (0)^2}$$

$$VT = \sqrt{36}$$

Two sides are congruent. So, ΔTUV is isosceles.

Step 2 Add the lengths of the sides to find the perimeter.

$$\overline{TU} + \overline{UV} + \overline{VT} = \sqrt{25} + \sqrt{25} + \sqrt{36}$$

$$= 5 + 5 + 6 \text{ or } 16 \text{ units}$$

Exercises

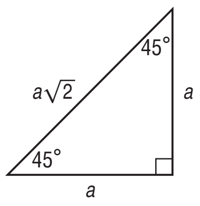
1. Classify ΔABC with vertices A $(-5, 3)$, B $(2, 4)$, and C $(1, -4)$ by its sides. Then find its perimeter to the nearest tenth.

2. Classify ΔGHI with vertices G $(-2, -5)$, H $(2, 3)$, and I $(6, -5)$ by its sides. Then find its perimeter to the nearest tenth.

10-6 Study Guide and Intervention

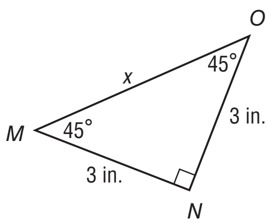
Special Right Triangles

Find Measures in 45°-45°-90° Triangles A 45°-45°-90° triangle is a special right triangle whose angles measure 45°, 45°, and 90°, creating a right isosceles triangle. All 45°-45°-90° triangles are similar. They have corresponding, congruent angles and proportional side lengths.

45°-45°-90° Triangles			
Words	In a 45°-45°-90° triangle, the length of the hypotenuse is $\sqrt{2}$ times the length of a leg.	**Model**	
Symbols	hypotenuse = leg $\cdot \sqrt{2}$		

Example **Find the length of each hypotenuse.**

a.



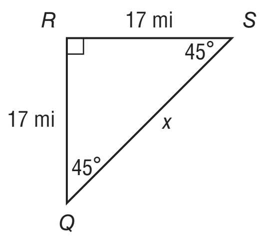
$c = a \cdot \sqrt{2}$ Relationship for a 45°-45°-90° triangle

$c = 3 \cdot \sqrt{2}$ Replace a with 3.

$c = 3\sqrt{2}$ Simplify.

The hypotenuse measures $3\sqrt{2}$ inches.

b.



$c = a \cdot \sqrt{2}$ Relationship for a 45°-45°-90° triangle

$c = 17 \cdot \sqrt{2}$ Replace a with 17.

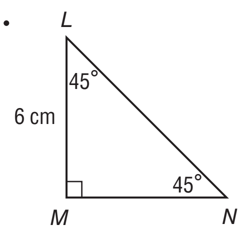
$c = 17\sqrt{2}$ Simplify.

The hypotenuse measures $17\sqrt{2}$ miles.

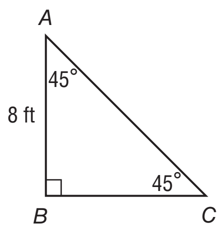
Exercises

Find the length of each hypotenuse.

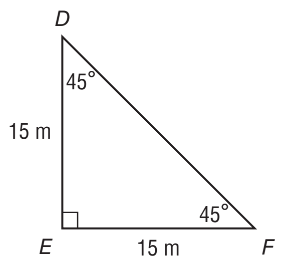
1.



2.



3.



10-6 Study Guide and Intervention *(continued)*

Special Right Triangles

Find Measures in 30°-60°-90° Triangles Another special right triangle is a 30°-60°-90° triangle. Just as all 45°-45°-90° triangles are similar, all 30°-60°-90° triangles are similar. They have corresponding, congruent angles and proportional side lengths.

30°-60°-90° Triangles

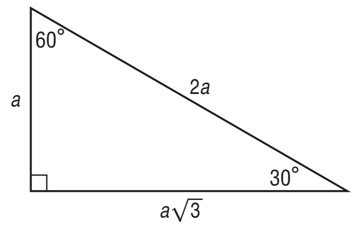
Words In a 30°-60°-90° triangle,

- the length of the hypotenuse is 2 times the length of the shorter leg, and
- the length of the longer leg is $\sqrt{3}$ times the length of the shorther leg.

Symbols hypotenuse = 2 · shorter leg

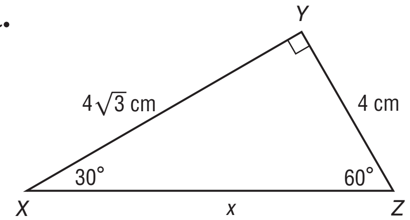
longer leg = $\sqrt{3}$ · shorter leg

Model



Example **Find the length of each missing measure.**

a.



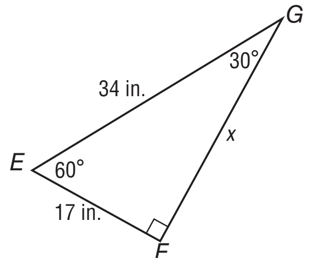
$c = 2a$ Relationship for a 30°-60°-90° triangle

$c = 2(4)$ Replace a with 4.

$c = 8$ Simplify.

The hypotenuse measures 8 centimeters.

b.



$b = a \cdot \sqrt{3}$ Relationship for a 30°-60°-90° triangle

$b = 17 \cdot \sqrt{3}$ Replace a with 17.

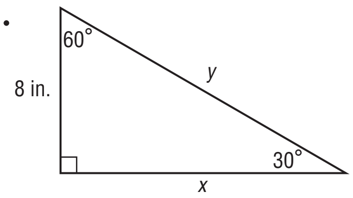
$b = 17\sqrt{3}$ Simplify.

The longer leg measures $17\sqrt{3}$ inches.

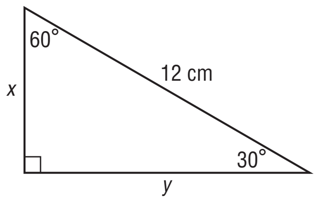
Exercises

Find the length of each missing measure.

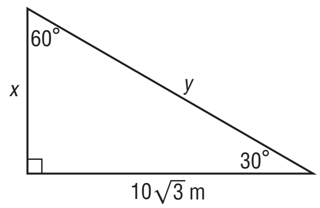
1.



2.



3.



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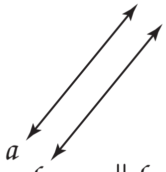
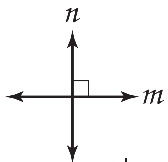
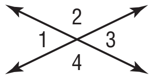
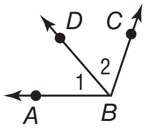
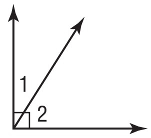
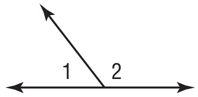
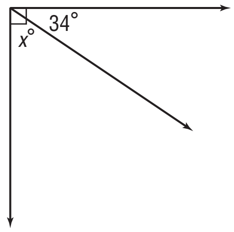
11-1 Study Guide and Intervention

Angle and Line Relationships

Line and Angle Relationships					
Parallel Lines	**Perpendicular Lines**	**Vertical Angles**	**Adjacent Angles**	**Complementary Angles**	**Supplementary Angles**
 $a \parallel b$	 $m \perp n$	 $\angle 1 \cong \angle 3$ $\angle 2 \cong \angle 4$	 $m\angle ABC =$ $m\angle 1 + m\angle 2$	 $m\angle 1 + m\angle 2 = 90°$	 $m\angle 1 + m\angle 2 = 180°$

Example **In the figure at the right, classify the relationship between the pairs of angles shown. Then find the value of *x*.**



The angles are complementary. The sum of their measures is 90°.

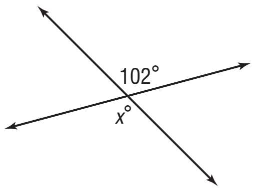
$$m\angle x + 34 = 90 \quad \text{Write the equation.}$$
$$m\angle x + 34 - 34 = 90 - 34 \quad \text{Subtract 34 from each side.}$$
$$m\angle x = 56 \quad \text{Simplify.}$$

So, $m\angle x$ is 56°.

Exercises

Classify the pairs of angles shown. Then find the value of *x* in each figure.

1.



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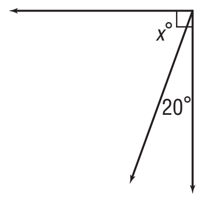
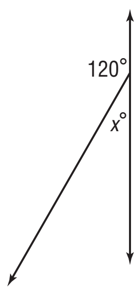
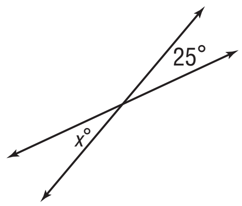


3.



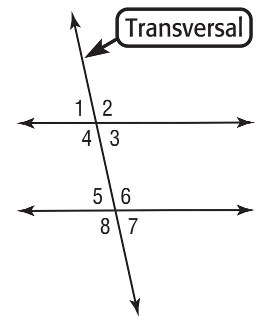
4.



11-1 Study Guide and Intervention *(continued)*

Angle and Line Relationships

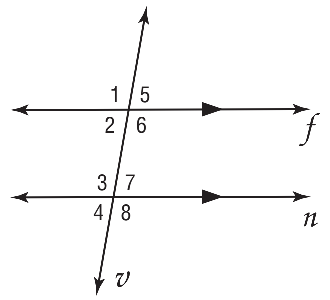
Names of Special Angles	
Interior angles lie inside the parallel lines.	∠3, ∠4, ∠5, ∠6
Exterior angles lie outside the parallel lines.	∠1, ∠2, ∠7, ∠8
Alternate interior angles are on opposite sides of the transversal and inside the parallel lines.	∠3 and ∠5, ∠4 and ∠6
Alternate exterior angles are on opposite sides of the transversal and outside the parallel lines.	∠1 and ∠7, ∠2 and ∠8
Corresponding angles are in the same position on the parallel lines in relation to the transversal.	∠1 and ∠5, ∠2 and ∠6, ∠3 and ∠7, ∠4 and ∠8



When a transversal intersects two parallel lines, pairs of alternate exterior angles, alternate interior angles, and corresponding angles are congruent.

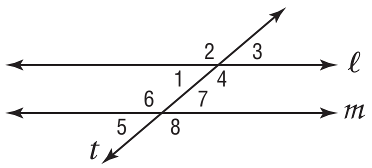
Example **In the figure, $f \parallel n$ and v is a transversal.**
If $m\angle 3 = 100°$, find $m\angle 1$ and $m\angle 6$.

Since ∠1 and ∠3 are corresponding angles, they are congruent. So, $m\angle 1 = 100°$. Since ∠3 and ∠6 are alternate interior angles, they are congruent. So, $m\angle 6 = 100°$.



Exercises

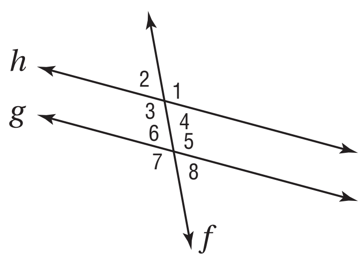
In the figure on the right, $l \parallel m$ and t is a transversal. If $m\angle 1 = 61.2°$ and the $m\angle 6 = 118.8°$, find the measure of each angle.



1. ∠7 **2.** ∠3 **3.** ∠4

4. ∠8 **5.** ∠5 **6.** ∠2

In the figure on the right, $g \parallel h$ and f is a transversal. If $m\angle 1 = 125°$ and the $m\angle 6 = 55°$, find the measure of each angle.

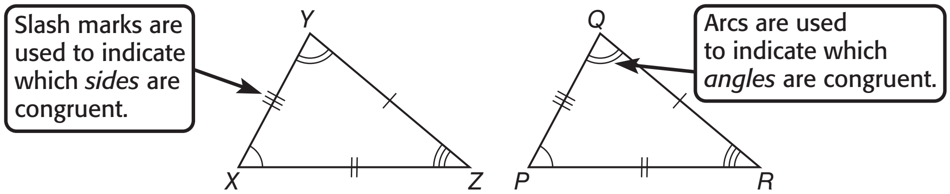


7. ∠2 **8.** ∠4 **9.** ∠5

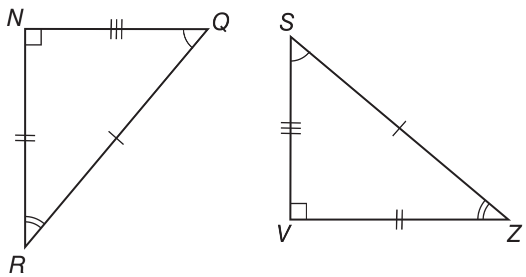
10. ∠3 **11.** ∠8 **12.** ∠7

11-2 Study Guide and Intervention

Congruent Triangles

Corresponding Parts of Congruent Triangles	
Words	Two triangles are **congruent** if they have the same size and shape. If two triangles are congruent, their corresponding sides are congruent and their corresponding angles are congruent.
Model	
Symbols	Congruent Angles: $\angle X \cong \angle P$, $\angle Y \cong \angle Q$, $\angle Z \cong \angle R$ Congruent Sides: $\overline{XY} \cong \overline{PQ}$, $\overline{YZ} \cong \overline{QR}$, $\overline{XZ} \cong \overline{PR}$

Example **Name the corresponding parts in the congruent triangles shown. Then write a congruence statement.**



Corresponding angles:

$\angle Q \cong \angle S$, $\angle R \cong \angle Z$, $\angle N \cong \angle V$

Corresponding sides:

$\overline{SZ} \cong \overline{QR}$, $\overline{ZV} \cong \overline{RN}$, $\overline{VS} \cong \overline{NQ}$

$\Delta NQR \cong \Delta VSZ$

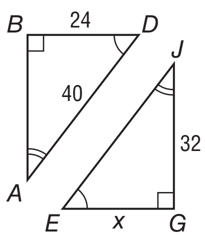
Exercises

Complete each congruence statement if $\Delta DFH \cong \Delta PWZ$.

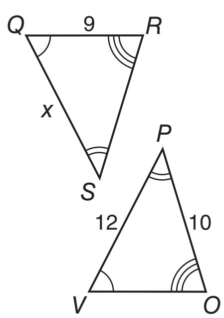
1. $\angle F \cong$ _____ **2.** $\angle P \cong$ _____ **3.** $\overline{DH} \cong$ _____ **4.** $\overline{ZW} \cong$ _____

Find the value of *x* for each pair of congruent triangles.

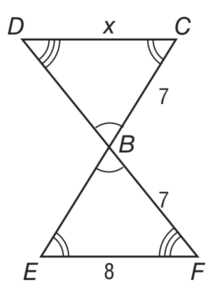
5.



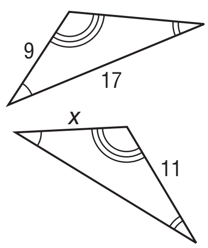
6.



7.



8.



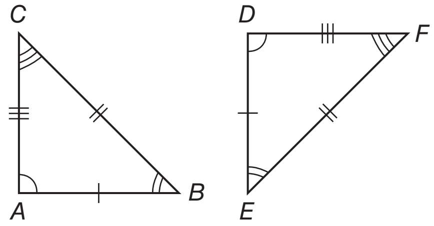
11-2 Study Guide and Intervention *(continued)*

Congruent Triangles

Identify Congruent Triangles Two triangles are congruent if and only if all pairs of corresponding angles are congruent and all pairs of corresponding sides are congruent.

Example **Determine whether the triangles shown are congruent. If so, name the corresponding parts and write a congruence statement.**

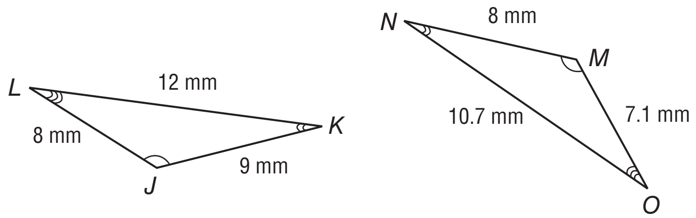
a.



Corresponding angles: The arcs indicate that $\angle A \cong \angle D$, $\angle B \cong \angle E$, and $\angle C \cong \angle F$.
Corresponding sides: The side measures indicate that $\overline{AB} \cong \overline{DE}$, $\overline{BC} \cong \overline{EF}$, and $\overline{CA} \cong \overline{FD}$.

Since all pairs of corresponding angles and sides are congruent, the triangles are congruent. One congruence statement is $\triangle ABC \cong \triangle DEF$.

b.

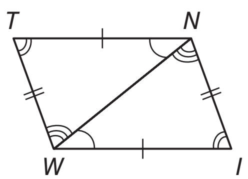


Although the arcs indicate that $\angle J \cong \angle M$, $\angle K \cong \angle N$, and $\angle L \cong \angle O$, the side measures indicate that no sides are congruent with one another. Therefore, the triangles are *not* congruent.

Exercises

Determine whether the triangles shown are congruent. If so, name the corresponding parts and write a congruence statement.

1.



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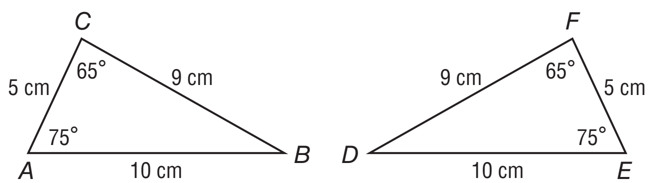
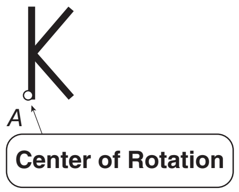


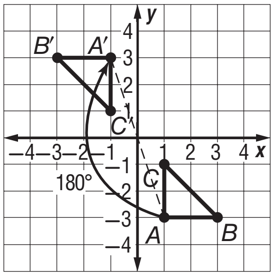
11-3 Study Guide and Intervention

Rotations

Rotations A **rotation** is a transformation in which a figure is turned around a fixed point. This point is called the **center of rotation.** A rotated figure has the same size and shape as the original figure.

Original Figure	Angle of Clockwise Rotation		
	90°	180°	270°
			

Example **Triangle *ABC* has vertices *A*(1, –3), *B*(3, –3), and *C*(1, –1). Graph the figure and its image after it is rotated 180° clockwise about the origin.**



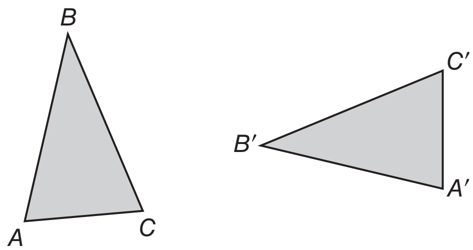
Step 1 Graph $\triangle ABC$ on a coordinate plane.

Step 2 Graph point A' after a 180° clockwise rotation about the origin.

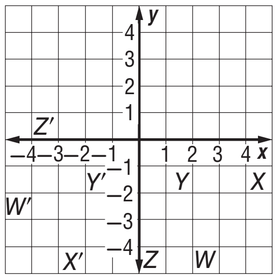
Step 3 Graph the remaining vertices after 180° rotations about the origin. Then connect the vertices to form $\triangle A'B'C'$.

Exercises

1. Draw the figure at the right after a 270° clockwise rotation about point *B*.



2. A figure has vertices *W*(2, –4), *X*(4, –2), *Y*(2, –2), and *Z*(0, –4). Graph the figure and its image after a clockwise rotation of 90° about the origin.

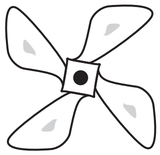


11-3 Study Guide and Intervention *(continued)*

Rotations

Rotational Symmetry A complete rotation of a figure is 360° because a circle has 360°. A figure that can be turned about its center less than 360° and match the original figure is said to have **rotational symmetry.** If the figure matches itself *only* after a 360° turn, it does not have rotational symmetry.

Example **TOYS Determine whether the pinwheel at the right has rotational symmetry. If it does, describe the angle of rotation.**



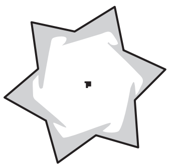
The pinwheel can match itself in four positions.

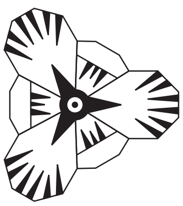
The pattern repeats in 4 even intervals.

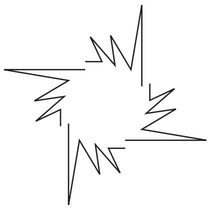
So, the angle of rotation is 360° ÷ 4 or 90°.

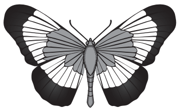
Exercises

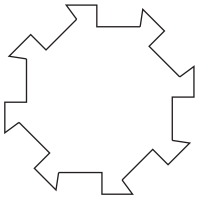
Determine whether each figure has rotational symmetry. If it does, describe the angle of rotation.

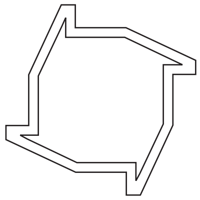
1. 

2. 

3. 

4. 

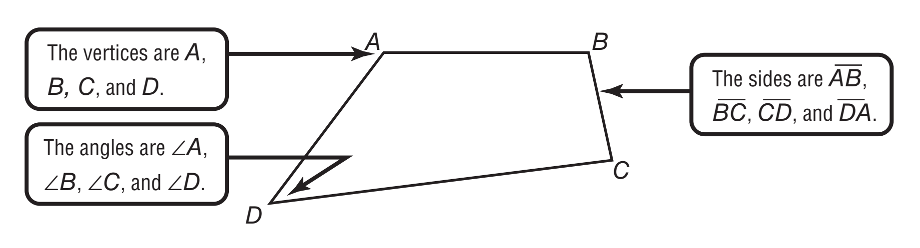
5. 

6. 

11-4 Study Guide and Intervention

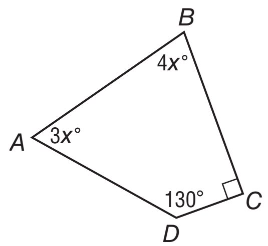
Quadrilaterals

A **quadrilateral** is a closed figure with four sides and four angles. The segments of a quadrilateral intersect only at their endpoints.



A quadrilateral can be separated into two triangles. The sum of the measures of the angles of a triangle is 180°. So, the sum of the measures of the angles of a quadrilateral is 2(180°) or 360°.

Example **Find the value of *x* in the quadrilateral. Then find each missing angle measure.**



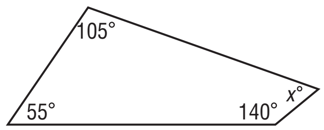
$3x + 4x + 90 + 130 = 360$	The sum of the angle measures is 360°.
$7x + 220 = 360$	Combine like terms.
$7x + 220 - 220 = 360 - 220$	Subtract 220 from each side.
$7x = 140$	Simplify.
$x = 20$	Divide each side by 7.

The value of x is 20. So, the missing angle measures are 3(20) or 60° and 4(20) or 80°.

Exercises

Find the value of *x* in each quadrilateral. Then find the missing angle measures.

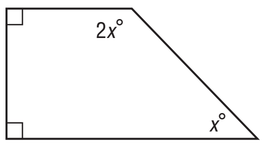
1.



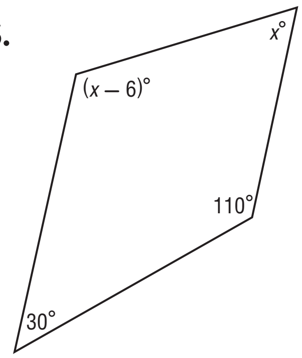
2.

3.

4.



5.



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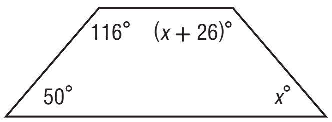


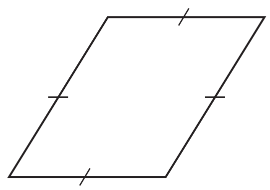
11-4 Study Guide and Intervention *(continued)*

Quadrilaterals

Classify Quadrilaterals Quadrilaterals can be classified by the relationship of their sides and angles.

- **Trapezoid** exactly one pair of parallel sides
- **Parallellogram** both pairs of opposite sides parallel and congruent
- **Rectangle** parallelogram with 4 right angles
- **Rhombus** parallelogram with 4 congruent sides
- **Square** parallelogram with 4 congruent sides and 4 right angles

Example **Classify the quadrilateral using the name that *best* describes it.**

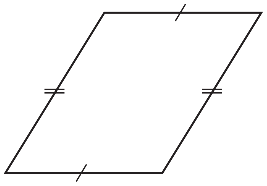


The opposite sides of the quadrilateral are parallel and all four sides are congruent. There are no right angles. It is a rhombus.

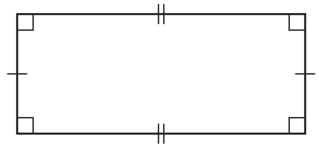
Exercises

Classify each quadrilateral using the name that *best* describes it.

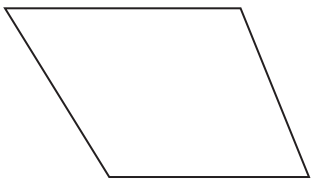
1.



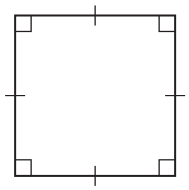
2.



3.



4.



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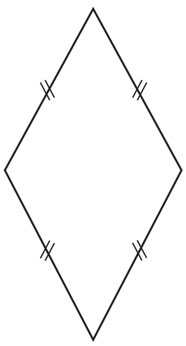

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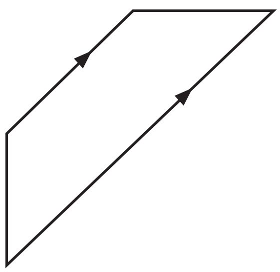


NAME ______________________________ DATE ____________ PERIOD ____________

11-5 Study Guide and Intervention

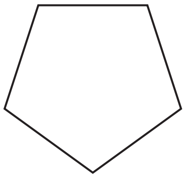
Polygons

Classify Polygons A **polygon** is a simple, closed figure formed by three or more coplanar line segments. The line segments, called *sides*, meet only at their endpoints. The points of intersection are called *vertices*. Polygons can be classified by the number of sides they have.

Number of Sides	Name of Polygon
3	triangle
4	quadrilateral
5	pentagon
6	hexagon
7	heptagon
8	octagon
9	nonagon
10	decagon

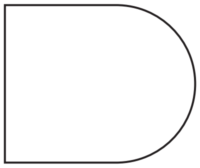
Example **Determine whether the figure is a polygon. If it is, classify the polygon. If it is not a polygon, explain why.**

a.



The figure has 5 sides that only intersect at their endpoints.
It is a pentagon.

b.

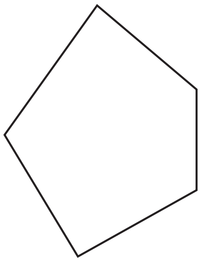


The figure has a curve. It is not a polygon.

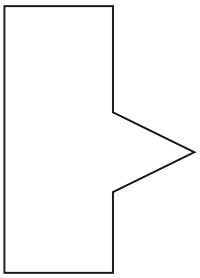
Exercises

Determine whether the figure is a polygon. If it is, classify the polygon. If it is not a polygon, explain why.

1.



2.



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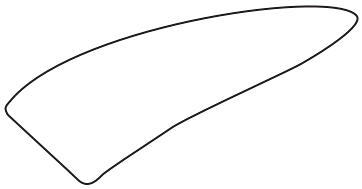
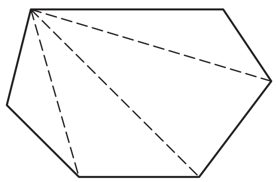


11-5 Study Guide and Intervention *(continued)*

Polygons

Find Angle Measures of a Polygon A **diagonal** is a line segment in a polygon that joins two nonconsecutive vertices, forming triangles. You can use the property of the sum of the measures of the angles of a triangle to find the sum of the measures of the interior angles of any polygon. An **interior angle** is an angle inside a polygon. A **regular polygon** is a polygon that is *equilateral* (all sides are congruent) and *equiangular* (all angles are congruent). Because the angles of a regular polygon are congruent, their measures are equal.

Interior Angles of a Polygon		
Words	If a polygon has n sides, then $n - 2$ triangles are formed. The sum of the degree measures of the interior angles of the polygon is $(n - 2)180$.	**Model** 
Symbols	$(n - 2)180$	6 sides → $n = 6$ 4 triangles

Example **Find the measure of one interior angle of a regular 20-gon.**

Step 1 A 20-gon has 20 sides. Therefore, $n = 20$.

$(n - 2)180 = (20 - 2)180$ Replace n with 20.

$= 18(180)$ or 3240 Simplify.

The sum of the measures of the interior angles is 3240°.

Step 2 Divide the sum by 20 to find the measure of one angle.

$3240 \div 20 = 162$

So, the measure of one interior angle in a regular 20-gon is 162°.

Exercises

Find the sum of the measures of the interior angles of each polygon.

1. quadrilateral **2.** nonagon **3.** heptagon **4.** hexagon

5. octagon **6.** pentagon **7.** decagon **8.** 12-gon

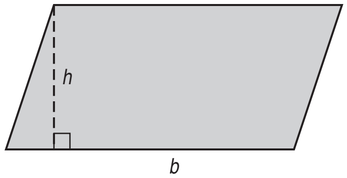
Find the measure of one interior angle of each polygon.

9. regular pentagon **10.** regular nonagon **11.** regular 18-gon

11-6 Study Guide and Intervention

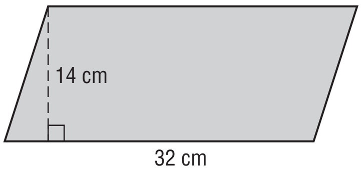
Area of Parallelograms, Triangles, and Trapezoids

Area of Parallelograms The **base** of a parallelogram is any side of the parallelogram. The *height* is the length of an **altitude,** a line segment perpendicular to the base with endpoints on the base and sides opposite the base.

Area of a Parallelogram		
Words	The area A of a parallelogram in square units is $A = bh$, where b is the base of the parallelogram and h is the height.	**Model** 
Symbols	$A = bh$	

Example **Find the area of each parallelogram.**

a.



Estimate $30 \cdot 14$ or 420

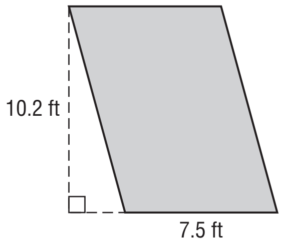
$A = bh$ — Area of a parallelogram

$= 32 \cdot 14$ — $b = 32$ and $h = 14$

$= 448$ — Multiply.

The area is 448 square centimeters. This is close to the estimate, 420, so the answer is reasonable.

b.



Estimate $8 \cdot 10$ or 80

$A = bh$ — Area of a parallelogram

$= 7.5 \cdot 10.2$ — $b = 7.5$ and $h = 10.2$

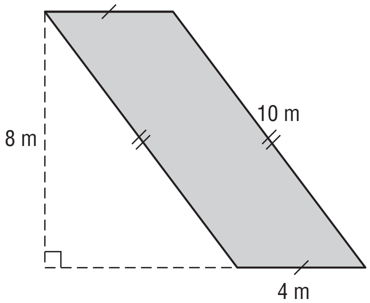
$= 76.5$ — Multiply.

The area is 76.5 square feet. This is close to the estimate, 80, so the answer is reasonable.

Exercises

Find the area of each parallelogram.

1.



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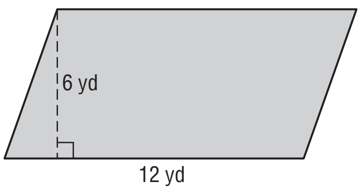

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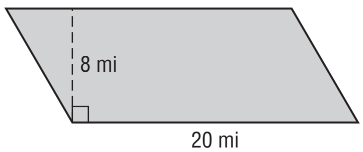

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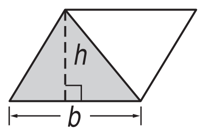
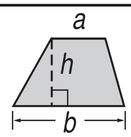
11-6 Study Guide and Intervention *(continued)*

Area of Parallelograms, Triangles, and Trapezoids

Shape	Words	Area Formula	Model
Triangle	A diagonal of a parallelogram separates the parallelogram into two congruent triangles. The area of each triangle is one-half the area of the parallelogram.	$A = \frac{1}{2}bh$	
Trapezoid	A trapezoid has two bases. The height of a trapezoid is the distance between the bases. A trapezoid can be separated into two triangles.	$A = \frac{1}{2}h(a + b)$	

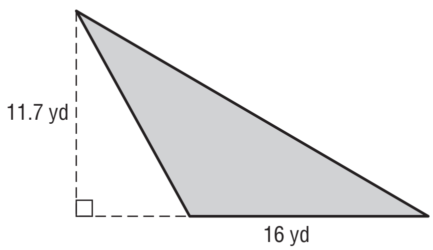
Example 1 **Find the area of the triangle.**

$A = \frac{1}{2}bh$ — Area of a triangle

$= \frac{1}{2}(16)(11.7)$ — $b = 16$ and $h = 11.7$

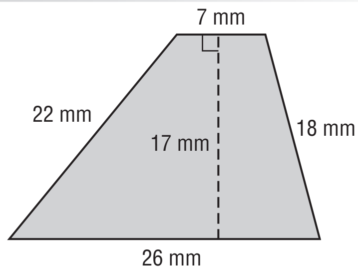
$= \frac{1}{2}(187.2)$ — Multiply $16 \cdot 11.7$.

$= 93.6$ — Simplify.



The area is 93.6 square yards.

Example 2 **Find the area of the trapezoid.**



$A = \frac{1}{2}h(a + b)$ — Area of a trapezoid

$= \frac{1}{2} \cdot 17(7 + 26)$ — Replace h with 17, a with 7, and b with 26.

$= \frac{1}{2} \cdot 17 \cdot 33$ — $7 + 26 = 33$

$= \frac{561}{2}$ or $280\frac{1}{2}$ — Simplify.

The area of the trapezoid is $280\frac{1}{2}$ square millimeters.

Exercises

Find the area of each figure.

1. triangle: height = 10 ft; base = 4 ft

2. trapezoid: height = 14 cm; bases = 8 cm, 5 cm

3. trapezoid: height = 9 in.; bases = 4 in., 2 in.

4. triangle: height = 14 ft; base = 7 ft

5. trapezoid: height = 16 m; bases = 9 m, 5 m

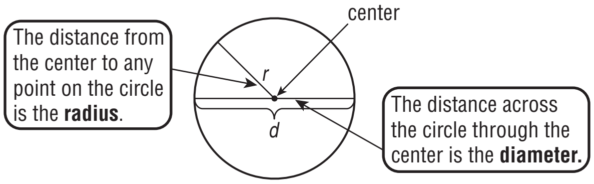
6. triangle: height = 8 yd; base = 12 yd

7. trapezoid: height = 15 mm; bases = 5 mm, 8 mm

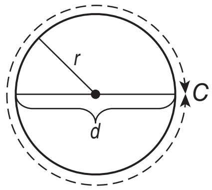
11-7 Study Guide and Intervention

Circles and Circumference

Circumference of Circles A **circle** is the set of all points in a plane that are the same distance from a given point, called the **center**.



The **circumference** of a circle is the distance around the circle. In every circle, the ratio of the circumference to the diameter is equal to approximately 3.14, represented by the Greek letter **π (pi).**

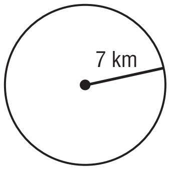
Circumference of a Circle		
Words	The circumference C of a circle is equal to its diameter times π, or 2 times its radius times π.	**Model** 
Symbols	$C = \pi d$ or $C = 2\pi r$	

Example **Find the circumference of the circle. Round to the nearest tenth.**

$C = 2\pi r$ — Circumference of a circle

$= 2 \cdot \pi \cdot 7$ — Replace r with 7.

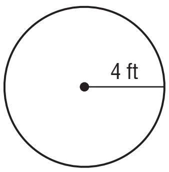
≈ 44.0 — Simplify. Use a calculator.



The circumference is about 44.0 kilometers.

Exercises

Find the circumference of each circle. Round to the nearest tenth.

1.


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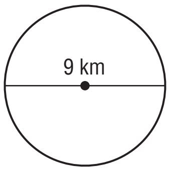

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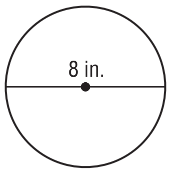

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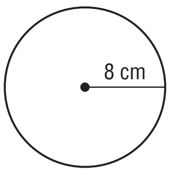
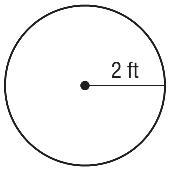
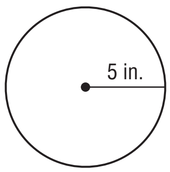


5.


6.


7. diameter = 5 centimeters

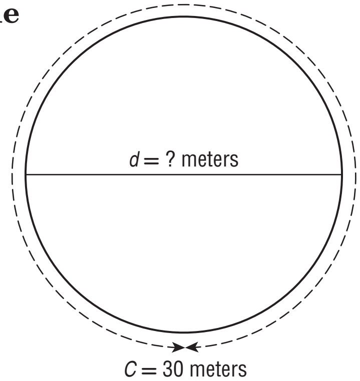
8. radius = 3 feet

11-7 Study Guide and Intervention *(continued)*

Circles and Circumference

Use Circumference to Solve Problems You can use circumference to solve real-world problems. If you know the circumference of a circle, you can determine the diameter or radius of the circle.

Example **GARDENS Arlene works at a retirement home that has a circular community garden with a circumference of 30 meters. She would like to use some edging to divide the garden down the center. What length of edging does she need?**



$C = \pi d$	Circumference of a circle
$30 = \pi \cdot d$	Replace *C* with 30.
$\frac{30}{\pi} = \frac{\pi d}{\pi}$	Divide each side by π.
$9.6 \approx d$	Simplify. Use a calculator.

So, the length of the edging should be about 9.6 meters.

Exercises

1. **BIKES** Bicycles are often classified by wheel diameter. A common diameter is 26 inches. What is the circumference of this bicycle tire? Round to the nearest tenth.

2. **FANS** The circular opening of a fan is 1 meter in diameter. What is the circumference of the circular opening of the fan?

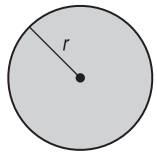
3. **FOUNTAINS** A circular fountain has a diameter of 10 meters. The statue in the middle of the fountain has a diameter of 1 meter. What is the circumference of the fountain?

4. **POOLS** You want to install a 1 yard wide walk around a circular swimming pool. The diameter of the pool is 20 yards. What is the distance around the outside edge of the walkway?

5. **TRAMPOLINES** The standard trampoline has a circumference of about 41 feet. When Jenna's dad lays with his feet at the center of the trampoline, the top of his head aligns with the outer edge. About how tall is Jenna's dad?

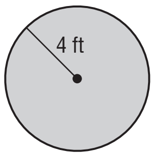
11-8 Study Guide and Intervention

Area of Circles

Area of Circles		
Words	The area A of a circle is equal to π times the square of its radius.	**Model** 
Symbols	$A = \pi r^2$	

Example **Find the area of each circle. Round to the nearest tenth.**

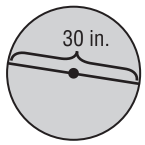
a.



$A = \pi r^2$	Area of a circle
$= \pi \cdot (4)^2$	Replace r with 4.
$= \pi \cdot 16$	Evaluate $(4)^2$.
≈ 12.56	Use a calculator.

The area is approximately 12.6 square feet.

b.

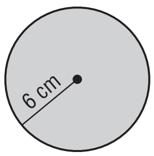


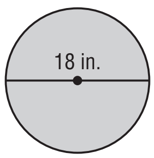
$A = \pi r^2$	Area of a circle
$= \pi \cdot (15)^2$	Replace r with 15.
$= \pi \cdot 225$	Evaluate $(15)^2$.
≈ 706.9	Use a calculator.

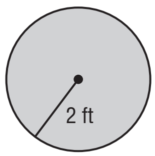
The area is about 706.9 square inches.

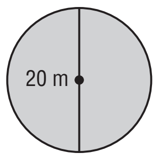
Exercises

Find the area of each circle. Round to the nearest tenth.

1. 

2. 

3. 

4. 

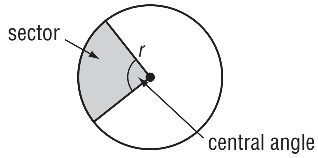
Match each circle described in the column on the left with its corresponding area in the column on the right.

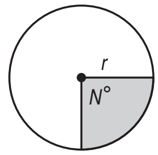
5. radius = 6 units	**a.** 452.2 units^2
6. diameter = 24 units	**b.** 803.8 units^2
7. diameter = 50 units	**c.** 1962.5 units^2
8. radius = 16 units	**d.** 113 units^2
9. diameter = 50 units	**e.** 2122.6 units^2
10. radius = 26 units	**f.** 1962.5 units^2

11-8 Study Guide and Intervention *(continued)*

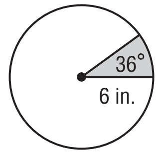
Area of Circles

Area of Sectors The area of a **sector** of a circle depends on the radius of the circle and the measure of the **central angle,** or the angle with a vertex at the center of the circle and with sides that intersect the circle.



Area of a Sector		
Words	The area A of a sector is $\frac{N}{360}(\pi r^2)$, where N is the degree measure of the central angle of the circle and r is the radius.	**Model** 
Symbols	$A = \frac{N}{360}(\pi r^2)$	

Example **Find the area of the shaded sector in the circle at the right. Round to the nearest tenth.**

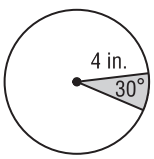


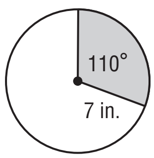
$A = \frac{N}{360}(\pi r^2)$	Area of a sector
$A = \frac{36}{360}(\pi)(6^2)$	Replace N with 36 and r with 6.
$= \frac{1}{10}(\pi)(36)$	Simplify.
≈ 11.3	Use a calculator.

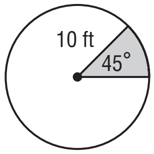
The area of the sector is about 11.3 square inches.

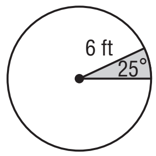
Exercises

Find the area of each shaded sector. Round to the nearest tenth.

1. 

2. 

3. 

4. 

5. The radius of a circle is 5 feet. It has a sector with a central angle of 54°. What is the area of the sector to the nearest tenth?

6. The diameter of a circle is 18 meters. It has a sector with a central angle of 48°. What is the area of the sector to the nearest tenth?

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11-9 Study Guide and Intervention

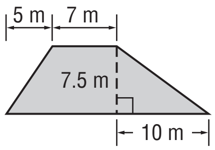
Area of Composite Figures

To find the area of a composite figure, decompose the composite figure into figures with area you know how to find. Use the area formulas you have learned in this chapter.

Triangle	Trapezoid	Parallelogram	Circle
$A = \frac{1}{2}bh$	$A = \frac{1}{2}h(a + b)$	$A = bh$	$A = \pi r^2$

Example **Find the area of each figure. Round to the nearest tenth, if necessary.**

a.



Area of Parallelogram

$A = bh$

$A = 7(7.5)$ or 52.5

Area of Triangle

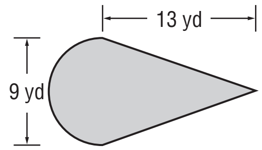
$A = \frac{1}{2}bh$

$A = \frac{1}{2}(15 \cdot 7.5)$

$A = 56.25$

The area of the figure is 52.5 + 56.25 or about 108.8 square meters.

b.



Area of Semicircle

$A = \frac{1}{2}\pi r^2$

$A = \frac{1}{2}\pi(4.5)^2$

$A = 31.8$

Area of Triangle

$A = \frac{1}{2}bh$

$A = \frac{1}{2}(9 \cdot 13)$

$A = 58.5$

The area of the figure is 31.8 + 58.5 or about 90.3 square yards.

Exercises

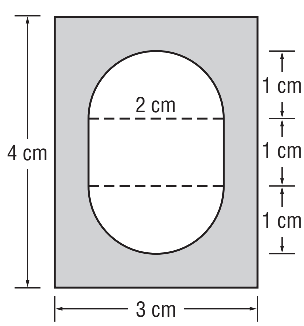
Find the area of each figure. Round to the nearest tenth, if necessary.

1. What is the area of a figure formed using a rectangle with a base of 10 yards and a height of 4 yards and two semicircles, one with a radius of 5 yards and the other a radius of 2 yards?

2. Find the area of a figure formed using a square and three triangles all with sides of 9 centimeters. Each triangle has a height of 6 centimeters.

Find the area of each shaded region. Round to the nearest tenth. (*Hint:* Find the total area and subtract the non-shaded area.)

3.



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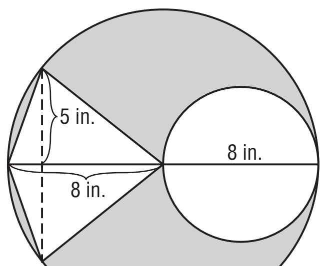

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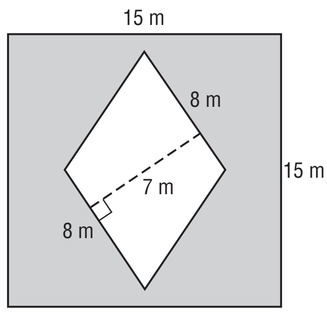
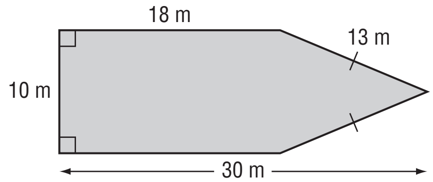


11-9 Study Guide and Intervention *(continued)*

Area of Composite Figures

Solving Problems Involving Area The area of a composite figure is calculated by dividing the composite figure into basic figures and then using the relevant area formula for each basic figure. Often the first step in a multi-step problem is to find the area of a composite figure.

Example **PARTIES Jonathon is renting a banquet hall to celebrate his 40th wedding anniversary. The cost to rent the hall is $5 per square meter. How much will Jonathon pay to rent the hall?**



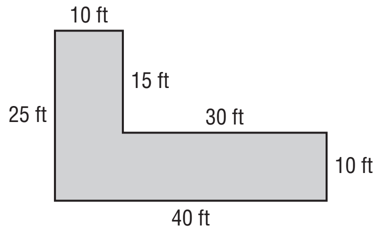
Separate the figure into a rectangle and a triangle. Find the sum of the areas of the figures.

$A = bh$	Area of rectangle	$A = \frac{1}{2}bh$	Area of triangle
$= 18 \cdot 10$	$b = 18$, $h = 10$	$= \frac{1}{2} \cdot 10 \cdot 12$	$b = 10$, $h = 12$
$= 180$	Simplify.	$= \frac{1}{2} \cdot 120$	Multiply.
		$= 60$	Simplify.

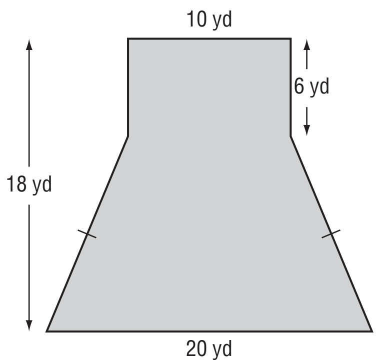
The area of the hall is 180 + 60 or 240 square meters. The cost to rent the hall is 240 · $5 or $1200.

Exercises

1. **LANDSCAPING** Deidre just purchased a new house and needs to landscape the yard. It will cost her $0.25 per square foot to cover the yard shown below with topsoil. How much will it cost Deidre to cover her yard in topsoil?



2. **CARPET** A restaurant owner wants to carpet his restaurant. The carpet costs $12 per square yard. Based on the floor plan below, how much will it cost him to carpet his restaurant?

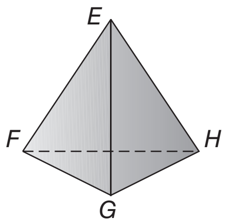


12-1 Study Guide and Intervention

Three-Dimensional Figures

Identify Three-Dimensional Figures A **prism** is a polyhedron with two parallel, congruent **bases**. A **pyramid** is a polyhedron with one base. Prisms and pyramids are named by the shape of their bases, such as triangular or rectangular.

Example 1 **Identify the figure. Name the bases, faces, edges, and vertices.**



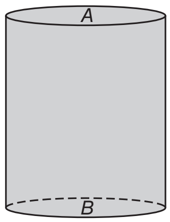
This figure has one triangular base, $\triangle FGH$, so it is a triangular pyramid.

faces: *EFG, EGH, EFH, FGH*

edges: $\overline{EF}, \overline{EG}, \overline{EH}, \overline{FG}, \overline{FH}, \overline{GH}$

vertices: *E, F, G, H*

Example 2 **Identify the figure. Name the bases, faces, edges, and vertices.**



This figure has two circular bases, *A* and *B*, so it is a cylinder.

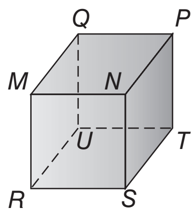
faces: *A* and *B*

The figure has no edges and no vertices.

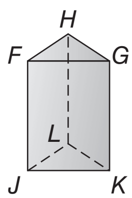
Exercises

Identify each figure. Name the bases, faces, edges, and vertices.

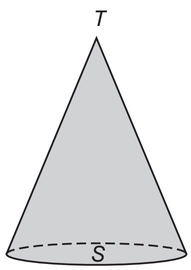
1.



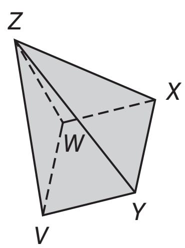
2.



3.



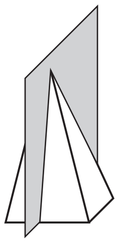
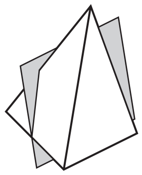
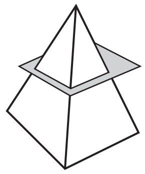
4.



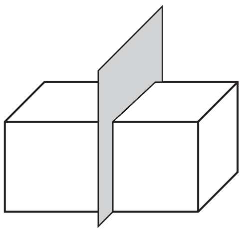
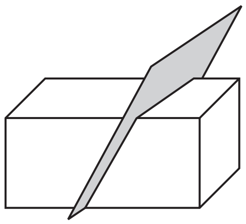
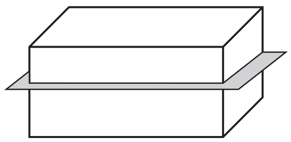
12-1 Study Guide and Intervention *(continued)*

Three-Dimensional Figures

Cross Sections When a plane intersects, or slices, a figure, the resulting figure is called a **cross section**. Figures can be sliced vertically, horizontally, or at an angle.

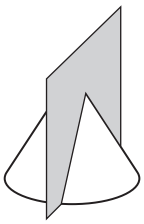
Vertical Slice	Angled Slice	Horizontal Slice
 This cross section is a triangle.	 This cross section is a trapezoid.	 This cross section is a square.

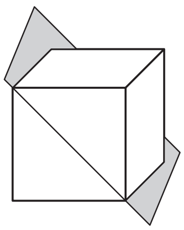
Example **Draw and describe the shape resulting from the following vertical, angled, and horizontal cross sections of a rectangular prism.**

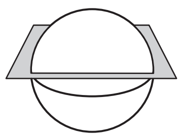
Vertical Slice	Angled Slice	Horizontal Slice
 This cross section is a rectangle.	 This cross section is a parallelogram.	 This cross section is a rectangle.

Exercises

Draw and describe the shape resulting from each cross section.

1. 

2. 

3. 

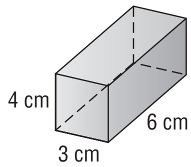
12-2 Study Guide and Intervention

Volume of Prisms

Volume of Prisms To find the volume V of a prism, use the formula $V = Bh$, where B is the area of the base, and h is the height of the solid.

Example **Find the volume of each prism.**

a.



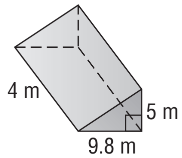
$V = Bh$

$V = (3 \cdot 6)4$

$V = 72$

The volume is 72 cm^3.

b.



$V = Bh$

$V = \left(\frac{1}{2} \cdot 9.8 \cdot 5\right)4$

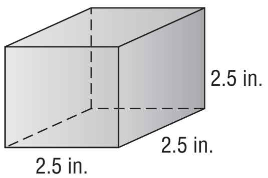
$V = 98$

The volume is 98 m^3.

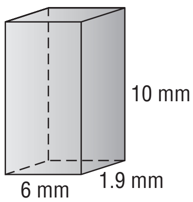
Exercises

Find the volume of each figure. If necessary, round to the nearest tenth.

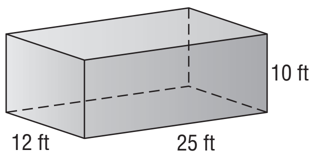
1.



2.



3.



4. Rectangular prism: length 9 millimeters, width 8.2 millimeters, height 5 millimeters

5. Triangular prism: base of triangle 5.8 feet, height of triangle 5.2 feet, height of prism 6 feet

6. Find the width of a rectangular prism with a length of 9 inches, a height of 6 inches, and a volume of 216 cubic inches.

7. Find the base length of a triangular prism with a triangle height of 8 feet, a prism height of 7 feet, and a volume of 140 cubic feet.

12-2 Study Guide and Intervention *(continued)*

Volume of Prisms

Volume of Composite Figures Figures that are made up of more than one type of figure are called composite figures. You can find the volume of a composite figure by breaking it into smaller components. Then, find the volume of each component and finally add the volumes of the components to find the total volume.

Example **TOYS Find the volume of the play tent at the right.**

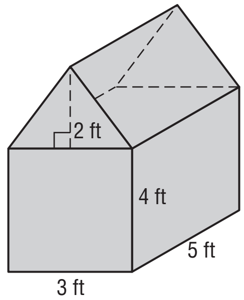
The figure is made up of a rectangular prism and a triangular prism. The volume of the figure is the sum of both volumes.

$V(\text{figure}) = V(\text{triangular prism}) + V(\text{rectangular prism})$

$V(\text{figure}) = Bh + \ell wh$ — Write the formulas for the volumes of the prisms.

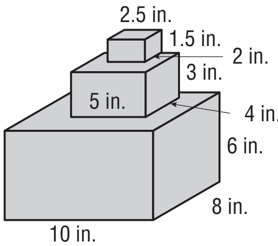
$= \frac{1}{2} \cdot 3 \cdot 2 \cdot 5 + 4 \cdot 3 \cdot 5$ — Substitute the appropriate values.

$= 15 + 60 \text{ or } 75 \text{ ft}^3$ — Simplify.

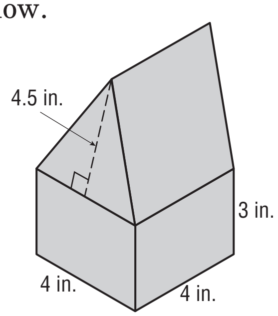


Exercises

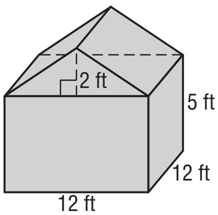
1. **GIFTS** Jamie made the tower of gifts shown below. Find the volume of the gifts.



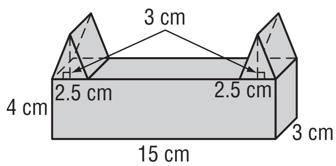
2. **GEOMETRY** Find the volume of the figure below.



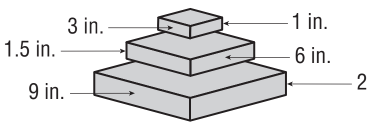
3. **TENTS** Mrs. Lyndon bought a patio tent. Find the volume of the tent.



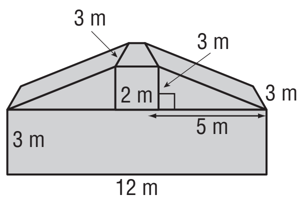
4. **MOLDS** Find the volume of the sandcastle mold shown below.



5. **PYRAMIDS** Ricky built a model of a square step pyramid. Find the volume of the pyramid.



6. **CANOPIES** Find the volume enclosed by the canopy shown below.

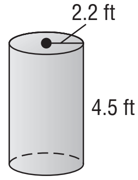


12-3 Study Guide and Intervention

Volume of Cylinders

Volumes of Cylinders Just as with prisms, the volume of a cylinder is based on finding the product of the area of the base and the height. The volume V of a cylinder with radius r is the area of the base, πr^2, times the height h, or $V = \pi r^2 h$.

Example 1 **Find the volume of the cylinder.**



$V = Bh$ Volume of a cylinder.

$V = \pi r^2 h$ Replace B with πr^2.

$\approx 3.14 \cdot 2.2^2 \cdot 4.5$ Replace π with 3.14, r with 2.2, and h with 4.5.

≈ 68.4 Simplify.

The volume is about 68.4 cubic feet.

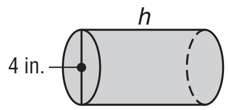
Check: You can estimate to check your work.

$V = \pi r^2 h \approx 3 \cdot 2^2 \cdot 5$ Replace π with 3, r with 2, and h with 5.

≈ 60 Simplify.

The estimate of 60 is close to the answer of 68.4. So, the answer is reasonable.

Example 2 **The volume of a cylinder is 150 cubic inches. Find the height of the cylinder. Round to the nearest whole number.**



$V = \pi r^2 h$ Volume of a cylinder.

$150 = 3.14 \cdot 2^2 \cdot h$ Replace V with 150, π with 3.14, and r with 2.

$150 = 12.56h$ Simplify.

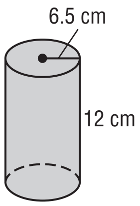
$12 \approx h$ Divide each side by 12.56. Round to the nearest whole number.

The height is about 12 inches.

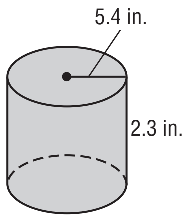
Exercises

Find the volume of each cylinder. Round to the nearest tenth.

1.

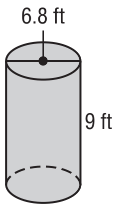


2.

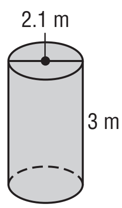


3. radius: 1.3 m
height: 3 m

4.



5.



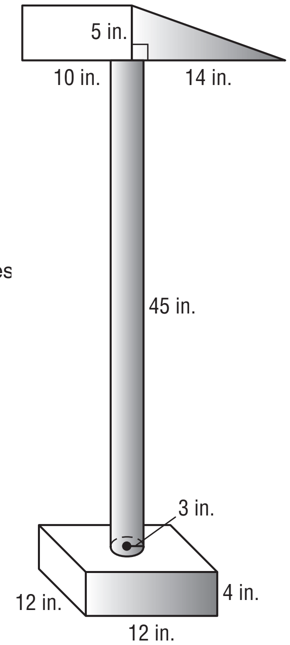
6. diameter: 11 cm
height: 6 cm

12-3 Study Guide and Intervention *(continued)*

Volume of Cylinders

Volumes of Composite Figures You can find the volume of composite figures with cylinders by separating the figure into the different pieces.

Example **PODIUMS A school principal ordered a podium for the debate club. Find the volume of the podium.**



The volume is the sum of the rectangular prism base, the cylindrical column, and the triangular prism top.

Step 1 Find the volume of the rectangular prism.

$V = Bh$ Volume of a prism

$V = 12 \cdot 12 \cdot 4$ The length and width are each 12 inches and the height is 4 inches

$= 576$ Simplify.

The volume of the rectangular prism base is 576 in^3.

Step 2 Find the volume of the cylinder.

$V = \pi r^2 h$ Volume of a cylinder

$V = 3.14 \cdot 3^2 \cdot 45$ Replace π with 3.14, *r* with 3, and *h* with 45.

≈ 1271.7 Simplify.

The volume of the cylinder is about 1271.7 in^3.

Step 3 Find the volume of the triangular prism.

$V = Bh$ Volume of a triangular prism

$V = \frac{1}{2} \cdot 14 \cdot 10 \cdot 5$ The length is 14, the width is 10, and the height is 5.

$= 350$ Simplify.

The volume of the triangular prism is 350 in^3.

Step 4 Find the volume of the composite figure.

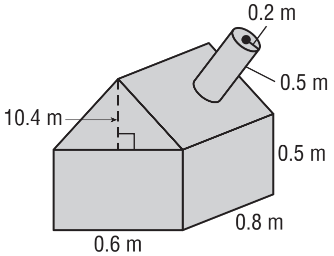
$576 + 1271.7 + 350 = 2197.7$

So, the total volume of the podium is 2197.7 in^3.

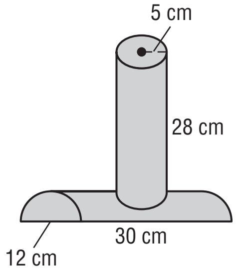
Exercises

Find the volume of each figure. Round to the nearest tenth.

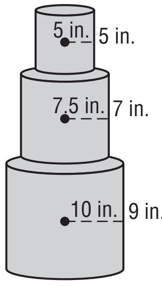
1.



2.



3.



NAME ______________________ DATE ____________ PERIOD ____________

12-4 Study Guide and Intervention

Volume of Pyramids, Cones, and Spheres

Volume of a Pyramid A pyramid has $\frac{1}{3}$ the volume of a prism with the same base and height. To find the volume V of a pyramid, use the formula $V = \frac{1}{3}Bh$, where B is the area of the base and h is the height of the pyramid.

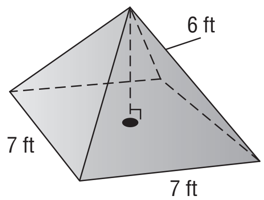
Example 1 **Find the volume of the pyramid.**

$V = \frac{1}{3}Bh$ Volume of a pyramid

$V = \frac{1}{3}(7 \cdot 7.6)$ The base is a square, so $B = 7 \cdot 7$. The height of the pyramid is 6 ft.

$V = 98$ Simplify.

The volume is 98 ft^3.



Volume of a Cone A cone has $\frac{1}{3}$ the volume of a cylinder with the same base and height. To find the volume V of a cone, use the formula $V = \frac{1}{3}\pi r^2 h$, where r is the radius and h is the height of the cone.

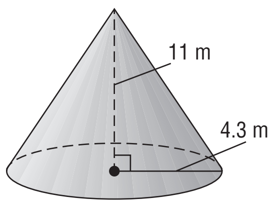
Example 2 **Find the volume of the cone. Round to the nearest tenth.**

$V = \frac{1}{3}\pi r^2 h$ Volume of a cone

$V = \frac{1}{3}\pi (4.3)^2 \cdot 11$ Replace r with 4.3 and h with 11.

$V \approx 213.0 \text{ m}^3$ Simplify. Round to the nearest tenth.

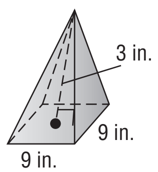
The volume is about 213.0 m^3.



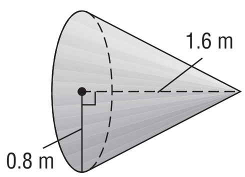
Exercises

Find the volume of each figure. Round to the nearest tenth, if necessary.

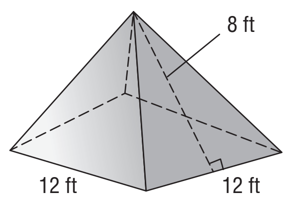
1.



2.



3.



4. Square pyramid: length 1.2 centimeters, height 5 centimeters

5. Cone: diameter 4 yards, height 7 yards

6. Rectangular prism: length 14.5 meters, width 5.2 meters, height 6.1 meters

12-4 Study Guide and Intervention *(continued)*

Volume of Pyramids, Cones, and Spheres

Volume of a Sphere To find the volume V of a sphere, use the formula $V = \frac{4}{3}\pi r^3$, where r is the radius.

Example 1 **Find the volume of the sphere. Round to the nearest tenth.**

$V = \frac{4}{3}\pi r^3$ Volume of a sphere

$V = \frac{4}{3}\pi(5)^3$ Replace r with 5.

$V \approx 523.6 \text{ in}^3$ Simplify.

The volume is about 523.6 in^3.

Example 2 **SOCCER A giant soccer ball has a diameter of 40 inches. Find the volume of the soccer ball. Then find how long it will take the ball to deflate if it leaks at a rate of 100 cubic inches per hour.**

Understand You know the diameter of the soccer ball.
You know the rate at which it is losing air.

Plan Find the volume of the ball.
Find how long it will take to deflate.

Solve $V = \frac{4}{3}\pi r^3$ Volume of a sphere

$= \frac{4}{3}\pi \cdot 20^3$ Since $d = 40$, replace r with 20.

$\approx 33{,}493.3 \text{ in}^3$ Simplify.

Use a proportion.

$$\frac{100 \text{ in}^3}{1 \text{ hour}} = \frac{33{,}493.3 \text{ in}^3}{x \text{ hour}}$$

$$100x = 33{,}493.3$$

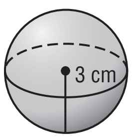
$$x \approx 334.9$$

So, it will take approximately 335 hours for the ball to deflate.

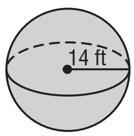
Exercises

Find the volume of each sphere. Round to the nearest tenth.

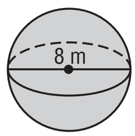
1.



2.



3.



4. Sphere: radius 5.2 miles

5. Sphere: diameter 11.6 feet

12-5 Study Guide and Intervention

Surface Area of Prisms

Lateral Area and Surface Area A prism consists of two parallel, congruent bases and a number of non-base faces. The non-base faces are called **lateral faces**. The **lateral area** of a figure is the sum of the areas of the lateral faces. The **surface area** of a figure is the total area of all the faces, or the sum of the lateral area plus the area of the bases.

To find the lateral area L of a prism with a height h and base with a perimeter P, use the formula $L = Ph$.

To find the surface area S of a prism with a lateral area L and a base area B, use the formula $S = L + 2B$. This can also be written as $S = Ph + 2B$.

Example 1 **Find the lateral and surface area of the rectangular prism.**

a. Find the lateral area.

$L = Ph$

$L = (2\ell + 2w)h$

$= (2 \cdot 2.1 + 2 \cdot 2.8)5.8$

$= 56.84 \text{ ft}^2$

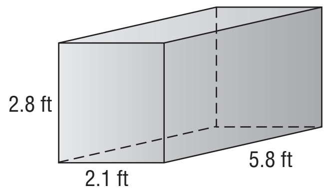
b. Find the surface area.

$S = L + 2B$

$S = L + 2\ell w$

$= 56.84 + 2 \cdot 2.1 \cdot 2.8$

$= 68.6 \text{ ft}^2$



Example 2 **Find the lateral and surface area of the triangular prism.**

a. Find the lateral area.

$L = Ph$

$= (5 + 5 + 6)7$

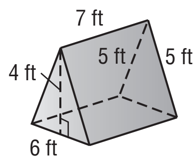
$= 112 \text{ ft}^2$

b. Find the surface area.

$S = L + 2B$

$S = 112 + 2 \cdot \frac{1}{2} \cdot 6 \cdot 4$

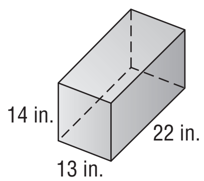
$= 136 \text{ ft}^2$



Exercises

Find the lateral and surface area of each prism. Round to the nearest tenth, if necessary.

1.



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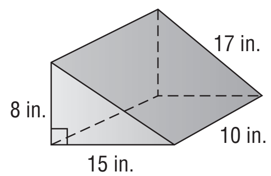

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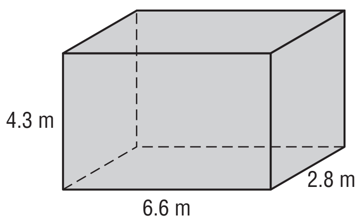

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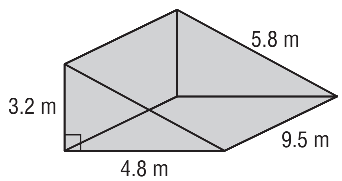


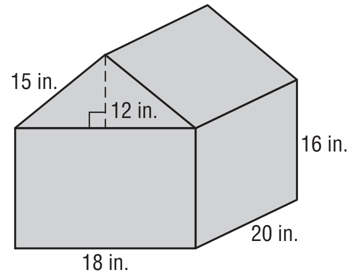
5. Cube: side length 8.3 centimeters

12-5 Study Guide and Intervention *(continued)*

Surface Area of Prisms

Problem Solving You can apply the formulas for lateral area and surface area to solve problems.

Example **CRAFTS Lena built a house out of cardboard. The roof is a triangular prism and the main part of the house is a rectangular prism. She wants to paint both parts before gluing them together. Find the amount of paint Lena needs if 1 ounce covers about 400 square inches.**



Triangular prism

a. Find the lateral area.

$L = Ph$

$= (15 + 15 + 18)20$

$= 960 \text{ in}^2$

b. Find the surface area.

$S = L + 2B$

$S = 960 + 2 \cdot \frac{1}{2} \cdot 18 \cdot 12$

$= 1176 \text{ in}^2$

Rectangular prism

a. Find the lateral area.

$L = Ph$

$L = (2\ell + 2w)h$

$= (2 \cdot 18 + 2 \cdot 16)20$

$= 1360 \text{ in}^2$

b. Find the surface area.

$S = L + 2B$

$S = L + 2\ell w$

$= 1360 + 2 \cdot 18 \cdot 16$

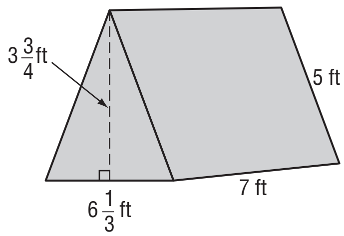
$= 1936 \text{ in}^2$

So, or the total area to be painted is 1176 + 1936 or 3112 in^2.
Since 3,112 ÷ 400 ≈ 7.75, Lena will need about 8 ounces of paint.

Exercises

1. **PAINTING** The walls of the school gym are being repainted. The gym is 50 feet long, 25 feet wide, and 16 feet high. Each wall will receive 2 coats of paint. If one gallon of paint covers 400 square feet, how many gallons are required?

2. **SPRAY-PAINTING** Kayla bought the tent shown at the right. She wants to spray all surfaces of the tent with waterproofing spray. Each 10-ounce bottle of spray will cover about 35 square feet. How many bottles of spray does Kayla need?



3. **PARTY FAVORS** For her birthday party, Rayna bought 12 boxes to decorate and give as party favors. She wants to decorate the boxes by covering them in fabric. Each box is a cube with side lengths of 5 inches. How many square inches of fabric does Rayna need?

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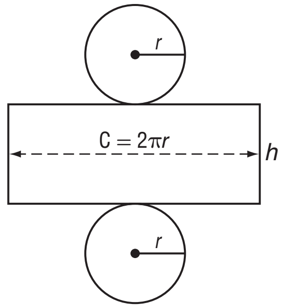
12-6 Study Guide and Intervention

Surface Area of Cylinders

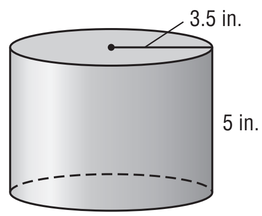
Surface Area of Cylinders As with a prism, the surface area of a cylinder is the sum of the lateral area and the area of the two bases. If you unroll a cylinder, its net is a rectangle (lateral area) and two circles (bases).



The lateral area L of a cylinder with radius r and height h is the product of the circumference of the base ($2\pi r$) and the height h. This can be expressed by the formula $L = 2\pi rh$.

The surface area S of a cylinder with a lateral area L and a base area B is the sum of the lateral area and the area of the two bases. This can be expressed by the formula $S = L + 2B$ or $S = 2\pi rh + 2\pi r^2$.

Example **Find the lateral and surface area of the cylinder.**



a. Find the lateral area.

$L = 2\pi rh$

$= 2 \cdot \pi \cdot 3.5 \cdot 5$

$= 35\pi \text{ in}^2$ exact answer

$\approx 109.9 \text{ in}^2$ approximate answer

b. Find the surface area.

$S = L + 2\pi r^2$

$= 35\pi + 2\pi(3.5)^2$

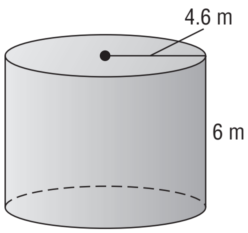
$= 59.5\pi \text{ in}^2$

$\approx 186.8 \text{ in}^2$

Exercises

Find the lateral and surface area of each cylinder. Round to the nearest tenth.

1.



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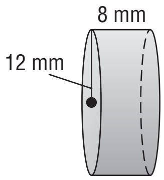

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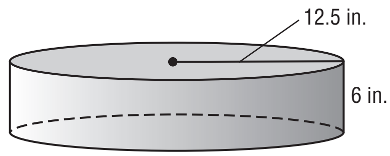

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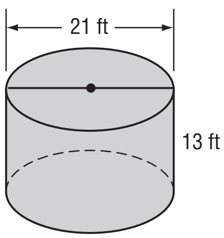

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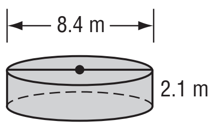

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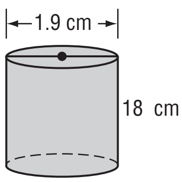


7. diameter of 20 yards and a height of 22 yards

8. radius of 7.6 centimeters and a height of 10.2 centimeters

12-6 Study Guide and Intervention *(continued)*

Surface Area of Cylinders

Problem Solving You can apply the formulas for lateral area and surface area to solve problems involving comparisons.

Example **DESIGN Marc studied package design in art class. He designed two cylindrical packages. One has a height of 4 inches and a diameter of 2.5 inches. The other has a height of 2.5 inches and a diameter of 4 inches. Which package has the greatest lateral area? Which has the greatest surface area?**

Step 1 Find the lateral area of both packages.

Lateral area of Package A
$L = 2\pi rh$
$= 2 \cdot \pi \cdot 1.25 \cdot 4$
$= 10\pi \text{ in}^2$
$\approx 31.4 \text{ in}^2$

Lateral area of Package B
$L = 2\pi rh$
$= 2 \cdot \pi \cdot 2 \cdot 2.5$
$= 10\pi \text{ in}^2$
$\approx 31.4 \text{ in}^2$

The lateral areas of the two packages are the same.

Step 2 Find the surface area of both packages.

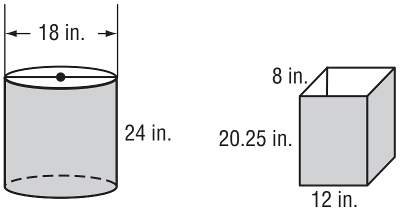
Surface area of Package A
$S = L + 2\pi r^2$
$= 10\pi + 2\pi(1.25)^2$
$= 13.125\pi \text{ in}^2$
$\approx 41.2 \text{ in}^2$

Surface area of Package B
$S = L + 2\pi r^2$
$= 10\pi + 2\pi(2)^2$
$= 18\pi \text{ in}^2$
$\approx 56.5 \text{ in}^2$

The surface area of Package B is greater than the surface area of Package A.

Exercises

1. **PAINTING** Gina is painting the garbage cans shown at the right. Both cans have the same volume. Which can has the greatest surface area? Explain.



2. **INSULATION** James is wrapping pipes in insulation. One pipe has a radius of 1.5 inches and a length of 30 inches. The other pipe has a radius of 3 inches and a length of 12.5 inches. Which pipe needs more insulation? Explain.

3. **STORAGE** There are two large cylindrical storage tanks at a factory. Both tanks are 12 feet high. One tank has a diameter of 8 feet and the other has a diameter of 16 feet. How does the surface area of the smaller tank relate to the surface area of the larger tank?

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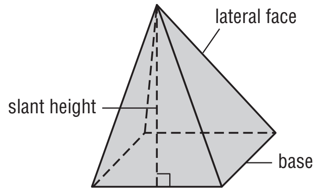
12-7 Study Guide and Intervention

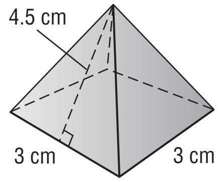
Surface Area of Pyramids and Cones



Surface Area of Pyramids **Regular pyramids** have bases which are a regular polygon and lateral faces which are congruent isosceles triangles. The height of each lateral face is called the **slant height** of the pyramid.

> The lateral area L of a regular pyramid is half the perimeter P of the base times the slant height ℓ or $L = \frac{1}{2}P\ell$. The total surface area S of a regular pyramid is the lateral area L plus the area of the base B or $S = L + B$, or $S = \frac{1}{2}P\ell + B$.

Example **Find the lateral and total surface area of the square pyramid.**



a. Find the lateral area.

$L = \frac{1}{2}P\ell$ Write the formula.

$L = \frac{1}{2}(3 \cdot 4)4.5$ Replace P with $3 \cdot 4$ and ℓ with 4.5.

$= 27 \text{ cm}^2$ Simplify.

b. Find the surface area.

$S = L + B$ Write the formula.

$S = 27 + (3 \cdot 3)$ Replace L with 27 and B with $3 \cdot 3$.

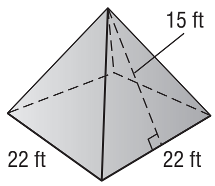
$= 36 \text{ cm}^2$ Simplify.

The lateral surface area is 27 cm^2, and the total surface area is 36 cm^2.

Exercises

Find the lateral and surface area of each regular pyramid. Round to the nearest tenth, if necessary.

1.



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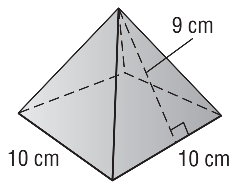

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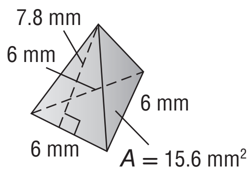

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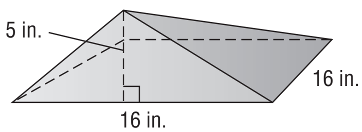

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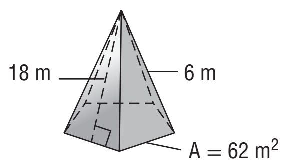

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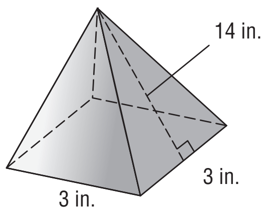


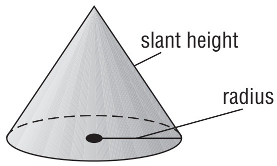
12-7 Study Guide and Intervention *(continued)*

Surface Area of Pyramids and Cones

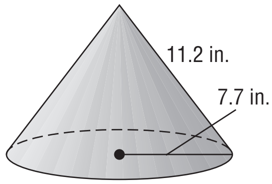
Surface Area of Cones

The lateral area L of a cone is the product of π, the radius r, and the slant height ℓ. This can be represented by the formula $L = \pi r \ell$.

The surface area S of a cone is the lateral area L plus the area of the base or πr^2. This can be represented by the formula $S = L + \pi r^2$.



Example **Find the lateral and total surface area of the cone. Round to the nearest tenth, if necessary.**



a. Find the lateral area.

$L = \pi r \ell$	Write the formula.
$L = \pi(7.7)(11.2)$	Replace r with 7.7 and ℓ with 11.2.
$\approx 270.8 \text{ in}^2$	Simplify.

b. Find the surface area.

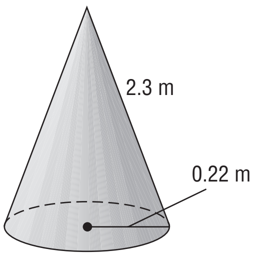
$S = L + \pi r^2$	Write the formula.
$S = 270.8 + \pi(7.7)^2$	Replace r with 7.7.
$\approx 457 \text{ in}^2$	Simplify.

The surface area is about 457 square inches.

Exercises

Find the lateral and surface area of each cone. Round to the nearest tenth, if necessary.

1.



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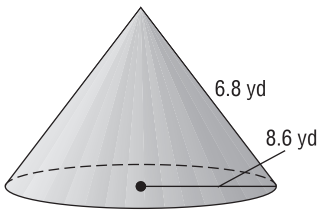

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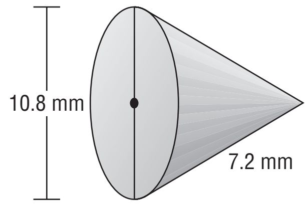

4. Cone: radius 7.2 meters, slant height 12 meters

5. Cone: diameter 16 inches, slant height 9 inches

6. Cone: diameter 5.5 yards, slant height 10 yards

7. Cone: diameter 3.6 feet, slant height 5.1 feet

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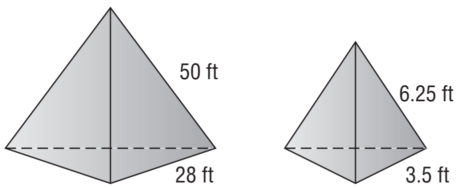
12-8 Study Guide and Intervention

Similar Solids

Identify Similar Solids Solids are similar if they have the same shape and their corresponding linear measures are proportional.

Example 1 **Determine whether each pair of solids is similar.**

a.



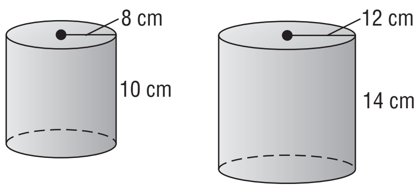
$\frac{50}{6.25} \stackrel{?}{=} \frac{28}{3.5}$ Write a proportion comparing corresponding edge lengths.

$50(3.5) \stackrel{?}{=} 6.25(28)$ Find the cross products.

$175 = 175$ ✓ Simplify.

The corresponding measures are proportional, so the pyramids are similar.

b.



$\frac{8}{12} \stackrel{?}{=} \frac{10}{14}$ Write a proportion comparing radii and heights.

$8(14) \stackrel{?}{=} 12(10)$ Find the cross products.

$112 \neq 120$ Simplify.

The radii and heights are not proportional, so the cylinders are not similar.

Use Similar Solids You can find missing measures if you know solids are similar.

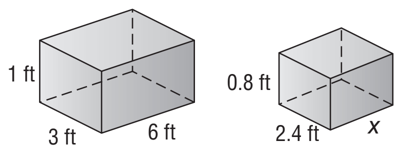
Example 2 **Find the missing measure for the pair of similar solids.**

$\frac{1}{0.8} = \frac{6}{x}$ Write a proportion.

$1x = 0.8(6)$ Find the cross products.

$x = 4.8$ Simplify.

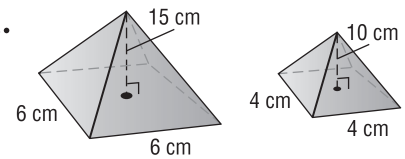
The missing length is 4.8 ft.



Exercises

Determine whether each pair of solids is similar.

1.



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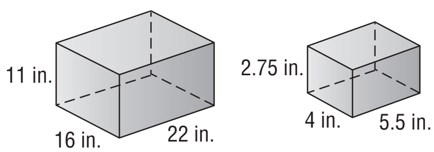
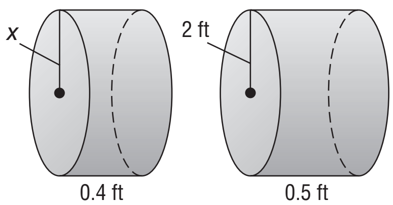


Find the missing measure for each pair of similar solids.

3.



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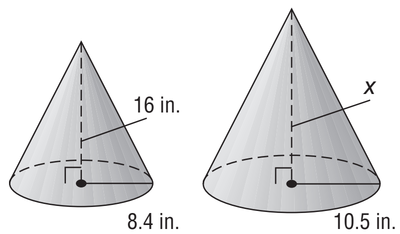

12-8 Study Guide and Intervention *(continued)*

Similar Solids

Properties of Similar Solids Just as corresponding sides of similar plane figures are proportional, corresponding linear measures of similar solids are proportional. The surface areas and volumes of similar solids are also related.

Ratio of Surface Area and Volume of Similar Solids

If two solids are similar with a scale factor of $\frac{a}{b}$, then the surface areas have a ratio $\left(\frac{a}{b}\right)^2$ and the volumes have a ratio $\left(\frac{a}{b}\right)^3$.

$\frac{\text{surface area of solid } A}{\text{surface area of solid } B} = \left(\frac{a}{b}\right)^2$ or $\frac{a^2}{b^2}$ $\qquad$ $\frac{\text{volume of solid } A}{\text{volume of solid } B} = \left(\frac{a}{b}\right)^3$ or $\frac{a^3}{b^3}$

Example **A triangular prism has surface area of 240 square inches and a volume of 120 cubic inches. If the dimensions are reduced by a factor of $\frac{1}{5}$, what is the surface area and volume of the new prism?**

Understand The prisms are similar and the scale factor of the side lengths $\frac{a}{b}$ is $\frac{1}{5}$.

Plan The surface area of the prisms have a ratio of $\frac{a^2}{b^2}$ or $\frac{1^2}{5^2}$. The volume of the prisms have a ratio of $\frac{a^3}{b^3}$ or $\frac{1^3}{5^3}$. Set up proportions to find the surface area and volume of the new prisms.

Solve

Surface Area

$\frac{S}{240} = \frac{1^2}{5^2}$ Let S = the surface area of the new prism.

$\frac{S}{240} = \frac{1}{25}$ $\quad \frac{1^2}{5^2} = \frac{1}{5} \cdot \frac{1}{5}$ or $\frac{1}{25}$

$S \cdot 25 = 240 \cdot 1$ Find the cross products.

$S = 9.6$ Divide each side by 25.

Volume

$\frac{V}{120} = \frac{1^3}{5^3}$ Let V = the volume of the new prism.

$\frac{V}{120} = \frac{1}{125}$ $\quad \frac{1^3}{5^3} = \frac{1}{5} \cdot \frac{1}{5} \cdot \frac{1}{5}$ or $\frac{1}{125}$

$V \cdot 125 = 120 \cdot 1$ Find the cross products.

$V = 0.96$ Divide each side by 125.

Exercises

1. A rectangular prism has a surface area of 130 square feet. If the dimensions are reduced by half, what is the surface area of the new prism?

2. A cone has a volume of 200.96 cubic feet. If the dimensions are tripled, what is the volume of the new cone?

3. **SCALE MODELS** The Great Pyramid in Giza, Egypt, has a square base with dimensions of 230 meters and a height of 147 meters. A model of the pyramid at a museum has a height of 2.94 meters. Find the scale factor between the actual pyramid and the model. Use this to find the area of the base of the model.

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13-1 Study Guide and Intervention

Measures of Central Tendency

Measures of Central Tendency When working with numerical data, it is often helpful to use one or more numbers to represent the whole set. These numbers are called the **measures of central tendency**. You will study the mean, median, and mode.

Statistic	Definition
mean	sum of the data divided by the number of items in the data set
median	middle number of the ordered data, or the mean of the middle two numbers
mode	number or numbers that occur most often

Example **Jason recorded the number of hours he spent watching television each day for a week. Find the mean, median, and mode for the number of hours.**

Mon.	Tues.	Wed.	Thurs.	Fri.	Sat.	Sun.
2	3.5	3	0	2.5	6	4

$$\text{mean} = \frac{\text{sum of hours}}{\text{number of days}}$$

$$= \frac{2 + 3.5 + 3 + \ldots + 4}{7} \text{ or } 3 \quad \text{The mean is 3 hours.}$$

To find the median, order the numbers from least to greatest and locate the number in the middle.

0 2 2.5 (3) 3.5 4 6 The median is 3 hours.

There is no mode because each number occurs once in the set.

Exercises

Find the mean, median, and mode for each set of data.

1. Maria's test scores

92, 86, 90, 74, 95, 100, 90, 50

2. Rainfall last week in inches

0, 0.3, 0, 0.1, 0, 0.5, 0.2

13-1 Study Guide and Intervention *(continued)*

Measures of Central Tendency

Choose Appropriate Measures To find the most appropriate measure of central tendency, examine each set of data for different criteria.

Measure	Most Useful When. . .
mean	• the data have no *extreme values* (values that are much greater or much less than the rest of the data)
median	• the data have extreme values • there are no big gaps in the middle of the data
mode	• the data have many repeated numbers

Example **BILLS The monthly grocery bill of three families was collected over 6 months. Which measure of central tendency best represents the data for each family?**

Pine	Kim	Diaz
\$310	\$210	\$204
\$143	\$254	\$187
\$324	\$210	\$195
\$153	\$193	\$214
\$311	\$214	\$416
\$169	\$210	\$146

Notice that the Pine's data has large gaps in the middle, so the median would not be an appropriate measure of central tendency. Mode would not be appropriate either since the data does not have any repeated numbers. The measure of central tendency that best represents the data for the Pine family would be the mean.

The Kim's data has three repeated numbers, \$210. The measure of central tendency that best represents the data for the Kim family would be the mode.

The Diaz's data has one extreme value and no repeated values. So, the measure of central tendency that best represents the data for the Diaz family would be the median.

Exercises

Find the measure of central tendency that best represents the data set(s).

A	B	C
1	8	5
33	2	25
3	3	8
4	11	10
8	9	5
5	10	4
9	8	5
6	2	6
2	9	8
7	3	5

1. A

2. B

3. C

4. A and B

5. A and C

6. B and C

NAME ______________________ DATE __________ PERIOD __________

13-2 Study Guide and Intervention

Stem-and-Leaf Plots

Stem-and-Leaf Plot

Words One way to organize and display data is to use a **stem-and-leaf plot**. In a stem-and-leaf plot, numerical data are listed in ascending or descending order.

Model

Stem	Leaf
2	0 1 1 2 3 5 5 6
3	1 2 2 3 7 9
4	0 3 4 8 8

3 | 7 = 37

The greatest place value of the data is used for the **stems**.

The next greatest place value forms the **leaves**.

Example **ZOOS Display the data shown at the right in a stem-and-leaf plot.**

Size of U. S. Zoos	
Zoo	**Size (acres)**
Audubon (New Orleans)	58
Cincinnati	85
Dallas	95
Denver	80
Houston	55
Los Angeles	80
Oregon	64
St. Louis	90
San Francisco	75
Woodland Park (Seattle)	92

Step 1 The least and the greatest numbers are 55 and 95. The greatest place value digit in each number is in the tens. Draw a vertical line and write the stems from 5 to 9 to the left of the line.

Stem	Leaf
5	8 5
6	4
7	5
8	5 0 0
9	5 0 2

Step 2 Write the leaves to the right of the line, with the corresponding stem. For example, for 85, write 5 to the right of 8.

Step 3 Rearrange the leaves so they are ordered from least to greatest. Then include a key or an explanation. Include a title.

U.S. Zoos

Stem	Leaf
5	5 8
6	4
7	5
8	0 0 5
9	0 2 5

8 | 5 = 85 acres

Exercises

Display each set of data in a stem-and-leaf plot.

1. {27, 35, 39, 27, 24, 33, 18, 19}

2. {94, 83, 88, 77, 95, 99, 88, 87}

3. {108, 113, 127, 106, 115, 118, 109, 112}

4. {64, 71, 62, 68, 73, 67, 74, 60}

13-2 Study Guide and Intervention *(continued)*

Stem-and-Leaf Plots

Interpret Data A stem-and-leaf plot can be very useful for analyzing data since the values are organized and easy to see. A **back-to-back stem-and-leaf plot** compares two sets of data side by side, with the leaves for one set of data on one side of the stem, and the leaves for the other set of data on the other side of the stem.

Example BOOKS **The number of books read by students in an eleventh-grade and a twelfth-grade English class are shown.**

Books Read by Students

Eleventh Grade	Stem	Twelfth Grade
8 7 6 5	0	3 6 8
7 7 5 5 4 3 2	1	3 3 6 7 8
1 0	2	2 2 5 6 7
6 2	3	6 9

1|2 = 21 books *1|8 = 18 books*

a. Find the median of each set of data.
The median of the eleventh-grade data is 15.
The median of the twelfth-grade data is 18.

b. What is the difference between the least number of books read and the most number of books read in each grade?
The greatest number of books read in the eleventh grade is 36 and in the twelfth grade is 39. The least number of books read in the eleventh grade is 5 and in the twelfth grade is 3. The difference between these numbers is $36 - 5$ or 31 for the eleventh grade, and $39 - 3$ or 36 for the twelfth grade.

c. In general, which class read the most books?
The twelfth-grade students read more books than the eleventh-grade students. There are more leaves in the 20 stem for the twelfth-grade data than there are for the eleventh-grade data.

d. Which grade has read a more varied number of books?
The twelfth-grade class has read a more varied number of books. The data for the eleventh-grade class is clustered in the 10 stem. The data for the twelfth-grade class is more spread out.

Exercises

COLLEGE **The stem-and-leaf plot on the right shows the number of college applications the students in two homeroom classes submitted.**

College Applications Submitted

Mr. Jones	Stem	Ms. Cho
9 8 6 5 5 3 2 2	0	0 0 0 1 1 2 2 3 4 5 5 5 6 7 9
8 7 5 4 4 4 3 2 1 0 0	1	4 5 6
6	2	2
	3	0

7|1 = 17 applications *3|0 = 30 applications*

1. Find the median of each set of data.

2. What is the difference between the least number of applications and the most number of applications in each class?

3. Which class submitted more applications?

4. Which class submitted a more varied number of applications?

13-3 Study Guide and Intervention

Measures of Variation

The **range** and the **interquartile range** describe how a set of data varies.

Term	Definition
range	The difference between the greatest and the least values of the set
median	The value that separates the data set in half
lower quartile	The median of the lower half of a set of data
upper quartile	The median of the upper half of a set of data
interquartile range	The difference between the upper quartile and the lower quartile
outlier	Data that are more than 1.5 times the value of the interquartile range beyond the quartiles

Example **Find the range, interquartile range, and any outliers for each set of data.**

a. {3, 12, 17, 2, 21, 14, 14, 8}

Step 1 List the data from least to greatest. The range is 21 − 2 or 19. Then find the median.

2 3 8 12 14 14 17 21

$\text{median} = \frac{14 + 12}{2}$ or 13

Step 2 Find the upper and lower quartiles.

2 3 8 12 14 14 17 21

$LQ = \frac{3 + 8}{2}$ or 5.5 median $UQ = \frac{14 + 17}{2}$ or 15.5

The interquartile range is 15.5 − 5.5 or 10. There are no outliers.

b.

Stem	Leaf
2	2 6 9
3	1 1 3 4 9
4	0 2 5 5 7 7 8
5	3 4 6 6

3 | 4 = 34%

The stem-and-leaf plot displays the data in order. The greatest value is 56. The least value is 22. So, the range is 56 − 22 or 34.

The median is 42. The LQ is 31 and the UQ is 48. So, the interquartile range is 48 − 31 or 17.

There are no outliers.

Exercises

WEATHER For Exercise 1, use the data in the stem-and-leaf plot at the right.

1. Find the range, median, upper quartile, lower quartile, interquartile range, and any outliers for each set of data.

Average Extreme July Temperatures in World Cities

Low Temps.		High Temps.
9 1 1 0	5	
4	6	4 7 9
9 8 6 5 5 4 3 0 0	7	9
0	8	1 1 3 3 4 8
	9	0 1 2 5
	10	7

0 | 8 = 80°F *7 | 9 = 79°F*

13-3 Study Guide and Intervention *(continued)*

Measures of Variation

Use Measures of Variation Measures of variation, just like measures of central tendency, can be used to compare and to interpret data.

Example **SONG LENGTHS The lengths in seconds of the last eighteen songs played on a radio station are shown. Use measures of variation to describe the data. Discuss how any outliers affect the measures of variation.**

Song Lengths

Stem	Leaf
11	0
12	
13	
14	3 5 6 7
15	2 2 4 6 7 8 8
16	2 2 4 5 8
17	
18	
19	
20	4

15|4 = 154 seconds

Find the measures of variation.

The range is 204 − 110 or 94.

The median is 156.5.

The lower quartile is 147.

The upper quartile is 162.

The interquartile range is 162 − 147 or 15.

There are two outliers, 110 and 204.

The songs are spread over 94 seconds. One fourth of the songs are 147 seconds or less. One fourth of the songs are 162 seconds or more. Half of the songs are between 147 and 162 seconds.

The two outliers, 110 and 204, affect the range since they are the largest and smallest values. They do not affect the median or the quartiles since they are at either end of the data set.

Exercises

MONEY RAISED For Exercise 1, use the data in the stem-and-leaf plot at the right.

1. Use the measures of variation to describe each data set.

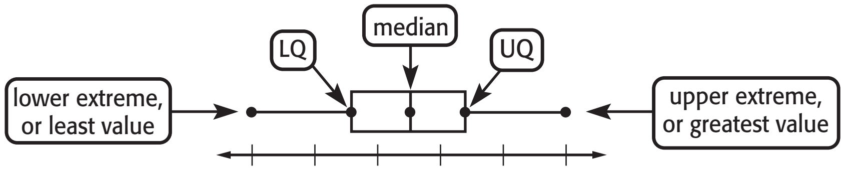
Amount of Money Raised for Field Trips by Each Student

History Club	Stem	Soccer Team
5 0	5	0 2 2 4 5
7 4 3	6	0
8 7 5 5 5	7	
8 5 3 3	8	2 3
	9	4 7
2	10	3 3 6 6 7

5|8 = $85 *9|4 = $94*

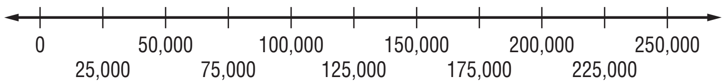
13-4 Study Guide and Intervention

Box-and-Whisker Plots

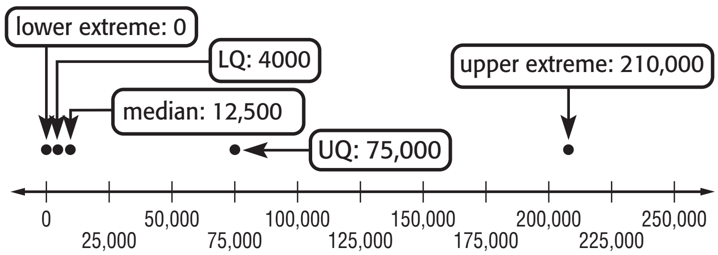
Box-and-Whisker Plot	**Words** A **box-and-whisker plot** divides a set of data into four parts using the median and quartiles. Each of these parts contains 25% of the data. **Model** 

Example **FOOD The heat levels of popular chile peppers are shown in the table. Display the data in a box-and-whisker plot.**

Step 1 Find the least and greatest number. Then draw a number line that covers the range of the data.

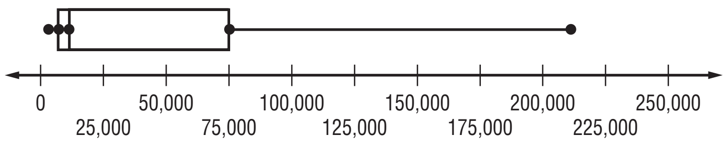


Step 2 Mark the median, the extremes, and the quartiles. Mark these points above the number line.



Step 3 Draw a box and the whiskers.

Heat Level of Chile Peppers



Heat Level of Chile Peppers

Name	Heat Level*
Aji escabeche	17,000
Bell	0
Cayenne	8,000
Habañero	210,000
Jalapeño	25,000
Mulato	1,000
New Mexico	4,500
Pasilla	5,500
Serrano	4,000
Tabasco	120,000
Tepín	75,000
Thai hot	60,000

Source: Chile Pepper Institute
*Scoville heat units

Exercises

Construct a box-and-whisker plot for each set of data.

1. {17, 5, 28, 33, 25, 5, 12, 3, 16, 11, 22, 31, 9, 11}

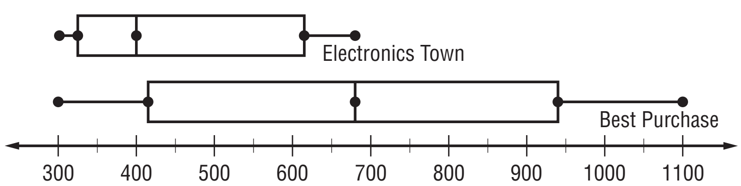
2. {$21, $50, $78, $13, $45, $5, $12, $37, $61, $11, $77, $31, $19, $11, $29, $16}

13-4 Study Guide and Intervention *(continued)*

Box-and-Whisker Plots

Intepret Box-and-Whisker Plots Although the four parts of a box-and-whisker plot may differ in length, each part still represents one-fourth, or 25%, of the data. A longer whisker or box shows the data have a greater range. A shorter whisker or box shows the data are more closely grouped together.

Example **COMPUTERS The price of computers in dollars at Electronics Town and Best Purchase are shown in the box-and-whisker plots below.**



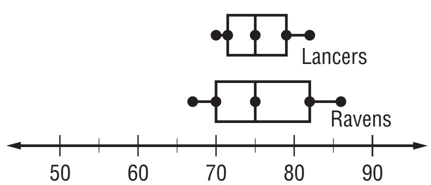
a. What percent of computers at Electronics Town cost less than $325?
At Electronics Town, 25% of the computers cost less than $350.

b. What percent of computers at Best Purchase cost less than $940?
At Best Purchase, 75% of the computers cost less than $940.

c. How does the price of computers at Electronics Town compare to the price of computers at Best Purchase?
Half of the computers at Best Purchase cost more than any computer at Electronics Town. The median price of the computers at Best Purchase is the same as the greatest price for computers at Electronics Town. The range of prices at Best Purchase is greater than the range of prices at Electronics Town. The prices of computers at Best Purchase are more varied than those at Electronics Town.

Exercises

HEIGHTS For Exercises 1–3, use the box-and-whisker plot which compares the heights of basketball players on two different teams.



1. What percent of the Ravens are 70 inches or taller?

2. What percent of the Lancers are 75 inches or taller?

3. How do the heights of the Ravens compare to the heights of the Lancers?

13-5 Study Guide and Intervention

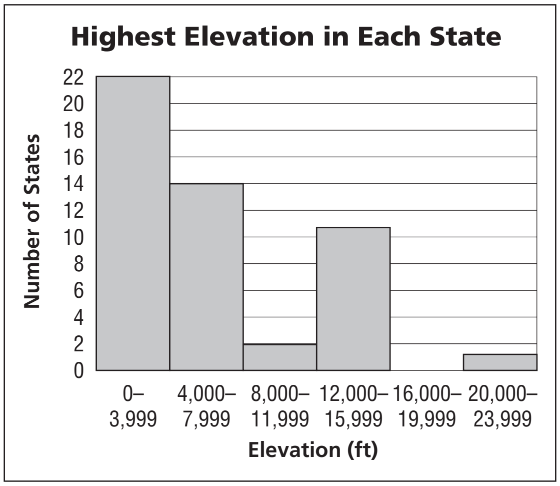
Histograms

Histograms	A histogram uses bars to display numerical data that have been organized into equal intervals. • There is no space between bars. • Because the intervals are equal, all of the bars have the same width. • Intervals with a frequency of 0 have no bar.

Example ELEVATIONS **The frequency table shows the highest elevations in each state. Display the data in a histogram.**

Highest Elevation in Each State		
Elevation (ft)	**Tally**	**Frequency**
0–3999	卌 卌 卌 卌 II	22
4000–7999	卌 卌 IIII	14
8000–11,999	II	2
12,000–15,999	卌 卌 I	11
16,000–19,999		0
20,000–23,999	I	1

Source: Peakware



Step 1 Draw and label the axes as shown. Include a title.

Step 2 Show the frequency intervals on the horizontal axis and an interval of 2 on the vertical axis.

Step 3 For each elevation interval, draw a bar whose height is given by the frequency.

Exercises

VOTING **For Exercise 1, use the information shown in the table below.**

1. The frequency table shows voter participation in a recent year. Display the data in a histogram.

Voter Participation by State		
Percent voting	**Tally**	**Frequency**
35–39	I	1
40–44		0
45–49	卌 I	6
50–54	卌 卌 II	12
55–59	卌 卌 III	13
60–64	卌 卌 II	12
65–69	卌 I	6

Source: U.S. Census Bureau

13-5 Study Guide and Intervention *(continued)*

Histograms

Interpret Data A histogram is a visual display of data in a frequency table, making it easier to interpret and compare the data.

Example **INTERNET The histogram at the right shows the number of hits the student Web sites in Ms. Foster's computer class get in a day.**

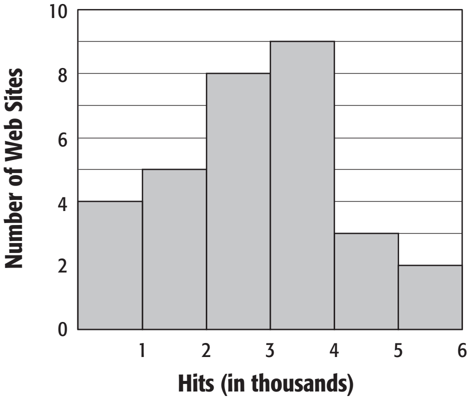
a. How many Web sites received 2,999 or less hits?

There were 4 + 5 + 8 or 17 student Web sites that received 2,999 or less hits.

b. What percent of Web sites received 5,000 or more hits?

There were 2 student Web sites that received 5,000 or more hits. There are a total of 4 + 5 + 8 + 9 + 3 + 2 or 31 students in Ms. Foster's class.

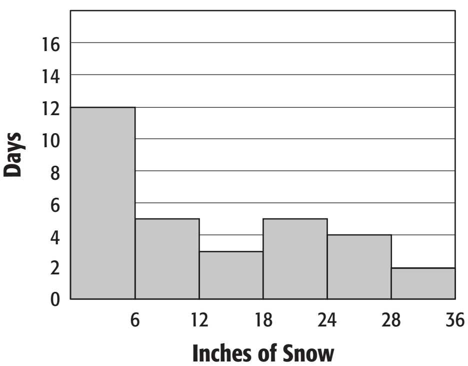
So $\frac{2}{31}$ or 6.5% of the Web sites received 5,000 or more hits.

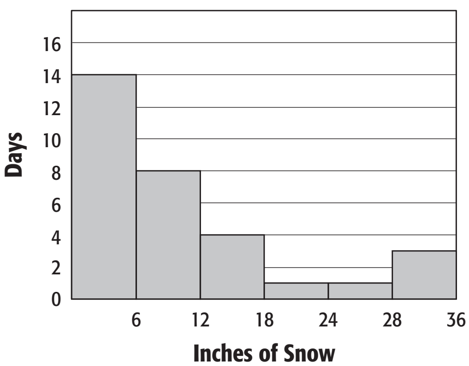


Exercises

SNOWFALL For Exercises 1–3, use the histograms below.

Inches of Snow in January





1. How many days did each city receive 12 or more inches of snow?

2. How many more days did Toronto receive 18 or more inches of snow than Seattle?

3. What was the greatest amount of snowfall for each city?

13-6 Study Guide and Intervention

Theoretical and Experimental Probability

You can measure the chance of an event happening with **probability.**
The **theoretical probability** is the chance that some event *should* happen.

$$P(\text{event}) = \frac{\text{number of favorable outcomes}}{\text{number of possible outcomes}}$$

The **experimental probability** is what *actually* happens when an experiment is repeated a number of times.

$$P(\text{event}) = \frac{\text{number of favorable outcomes that have happened}}{\text{number of outcomes that have happened}}$$

The **odds** in favor of an event is the ratio that compares the number of ways the event *can* occur to the number of ways that the event *cannot* occur. The **odds against** an event occuring is the ratio that compares the number of ways the event *cannot* occur to the number of ways that the event *can* occur.

Example 1 **A bag contains 6 red marbles, 1 blue marble, and 3 yellow marbles. One marble is selected at random. Find the theoretical probability of each outcome.**

a. P**(yellow)**

$$P(\text{event}) = \frac{\text{number of favorable outcomes}}{\text{number of possible outcomes}}$$

$$= \frac{3}{10} \text{ or } 30\%$$

There is a 30% chance of choosing a yellow marble.

b. P**(blue or yellow)**

$$P(\text{event}) = \frac{\text{number of favorable outcomes}}{\text{number of possible outcomes}}$$

$$= \frac{(1 + 3)}{10} = \frac{4}{10} \text{ or } 40\%$$

There is a 40% chance of choosing a yellow marble.

c. What are the odds in favor of picking a red marble?

Since there are 6 ways of picking a red marble, and 4 ways of not picking a red marble, the odds in favor are 6:4, or 3:2.

Example 2 **Ten marbles are selected from a bag of colored marbles. The results are shown in the table at the right. Find the experimental probability of selecting a red marble.**

Outcome	Frequency
Red	4
Blue	2
Yellow	4

$$P(\text{red}) = \frac{\text{number of favorable outcomes that have happened}}{\text{number of outcomes that have happened}}$$

$$= \frac{4}{10} \text{ or } 40\%$$

Exercises

A bag contains 5 red marbles, 5 blue marbles, 6 green marbles, 8 purple marbles, and 1 white marble. One is selected at random. Find the theoretical probability of each outcome. Express each theoretical probability as a fraction and as a percent.

1. P(white)

2. P(white, blue, or green)

3. P(red, blue, green, purple, or white)

13-6 Study Guide and Intervention *(continued)*

Theoretical and Experimental Probability

Use a Sample to Make Predictions To make a prediction about an event that will happen in the future, take a sample or survey of all the outcomes. Then use the experimental probability to predict how often that event will happen again.

Example **SHOES The chart to the right shows the number of people wearing different types of shoes in Mr. Thompson's English class. Suppose that there are 300 students in the cafeteria. Predict how many would be wearing low-top sneakers. Explain your reasoning.**

Shoes	Number of Students
Low-top sneakers	12
High-top sneakers	7
Sandals	3
Boots	6

Out of 12 + 7 + 3 + 6 or 28 students, 12 wore low-top sneakers.
So, you would expect $\frac{12}{28}$ or $\frac{3}{7}$ or about 43% of students to wear low-top sneakers.
Use the percent proportion to find 43% of 300.

$$\begin{matrix}\text{part} \rightarrow \\ \text{whole} \rightarrow\end{matrix} \frac{n}{300} = \frac{43}{100} \Big\} \text{ percent}$$

$100 \cdot n = 43 \cdot 300$

$100n = 12{,}900$ Find the cross products.

$= 129$ Mentally divide each side by 100.

Out of 300 students, you would expect about 129 students to wear low-top sneakers.

Exercises

DRIVERS From a survey of 100 drivers, 37 said they drove cars, 43 said they drove trucks, 12 said they drove vans, and 8 said they drove motorcycles. Out of 5,000 drivers, predict how many will drive the following vehicle(s).

1. car

2. truck

3. van or motorcycle

4. car or truck

5. truck or van

6. van or truck or car

INSURANCE An insurance company insures 2,342 homes. Of those homes, 1,234 are insured for fire, 456 are insured for fire and flood, and the rest are insured for flood. Out of 12,378 insured homes, predict how many will be insured for the following.

7. fire only

8. flood only

9. fire and flood

13-7 Study Guide and Intervention

Using Sampling to Predict

Identify Sampling Techniques A **sample** is a randomly selected smaller group chosen from the larger group, or **population**. An **unbiased sample** is representative of the larger population, selected without preference, and large enough to provide accurate data. A **biased sample** is not representative of the larger population.

Types of Unbiased Samples	
Type	**Definition**
Simple Random Sample	a sample where each item or person in a population is as likely to be chosen as any other
Stratified Random Sample	a sample in which the population is divided into similar, nonoverlapping groups. A simple random sample is then chosen from each group.
Systematic Random Sample	a sample in which the items or people are selected according to a specific time or item interval

Types of Biased Samples	
Type	**Definition**
Convenience Sample	a sample that includes members of the population that are easily accessed
Voluntary Response Sample	a sample which involves only those who want to or can participate in the sampling

Example **POLITICS To determine the popularity of a political candidate, 5 people are randomly polled from 10 different age groups of eligible voters. Identify the sample as biased or unbiased and describe its type.**

Since all eligible voters are equally likely to be polled, it is an unbiased sample. Since eligible voters are randomly polled from similar, non-overlapping groups, the sample is a stratified random sample.

Exercises

1. **STUDYING** To determine the average number of hours that students study, members of the math club are polled. Identify the sample as biased or unbiased and describe its type.

2. **TELEVISION** A television studio wants to know what viewers think about their programming. They mail a questionnaire to a random selection of residents in their area. Identify the sample as biased or unbiased and describe its type.

3. **POLITICS** A new bill is being passed in the state senate, but politicians want to know what their constituencies think. One politician goes to every 10th person's house in a neighborhood and asks how they feel about the bill. Identify the sample as biased or unbiased and describe its type.

4. **ENVIRONMENT** To test the frog population for diseases, an environmental group examines 50 males and 50 females. Identify the sample as biased or unbiased and describe its type.

13-7 Study Guide and Intervention *(continued)*

Using Sampling to Predict

Validating and Predicting Samples You can usually make predictions about the characteristics of larger populations based on a smaller sample of the population, depending on the method used to collect the sample.

Example 1 **SHOPPING To determine the number of first-time visitors to a mall, every 15th shopper to enter the mall was polled. There were 3000 total shoppers in the mall, and, of the shoppers polled, 26 shoppers were in the mall for the first time. Is this sampling method valid? If so, about how many of the 3000 shoppers were in the mall for the first time?**

Yes, this is a valid sampling method. This is a systematic random sample because the shoppers were selected according to a specific interval. Since every 15th shopper was sampled, there were a total of 3000 ÷ 15 or 200 shoppers sampled and 26 were in the mall for the first time. This means $\frac{26}{200}$ or 13% of the shoppers were in the mall for the first time. So a prediction of the total number of shoppers in the mall for the first time is 13% of 3000 or 390.

Example 2 **SUBSCRIPTIONS A magazine publisher mailed a survey to its subscribers to find out how many plan on renewing their subscriptions this year. Two hundred people responded that they would renew their subscriptions. Is this sampling method valid? If so, about how many of the 8000 subscribers will renew their subscriptions this year?**

This is a biased and voluntary response sample since it involves only those who want to participate in the survey. Only 2.5% (200 out of 8000) of the subscribers responded to the survey, so this is not an accurate or valid prediction of the total number of subscribers who will renew their subscriptions.

Exercises

1. **PRINTING** To determine the consistency of a printer, 100 printed sheets are randomly checked and 4 sheets are defective. What type of sampling method is this? About how many defective sheets would be expected if 2400 sheets were printed?

2. **MOVIES** A movie theater manager hands out surveys to 100 customers before the movie begins. At the end of the movie, 40 customers return their survey. Of the 40 surveys, 32 said they had a bad experience. What type of sampling method is this? Is this an accurate sampling method? If so, how many of the customers had a bad experience?

3. **QUALITY CONTROL** A TV manufacturing company wants to test the quality of their TVs. They randomly pick 50 TVs to test and determine that 4 are defective. What type of sampling method is this? About how many defective TVs would you expect if 1,000 TVs are made?

NAME ______________________________ DATE ____________ PERIOD ____________

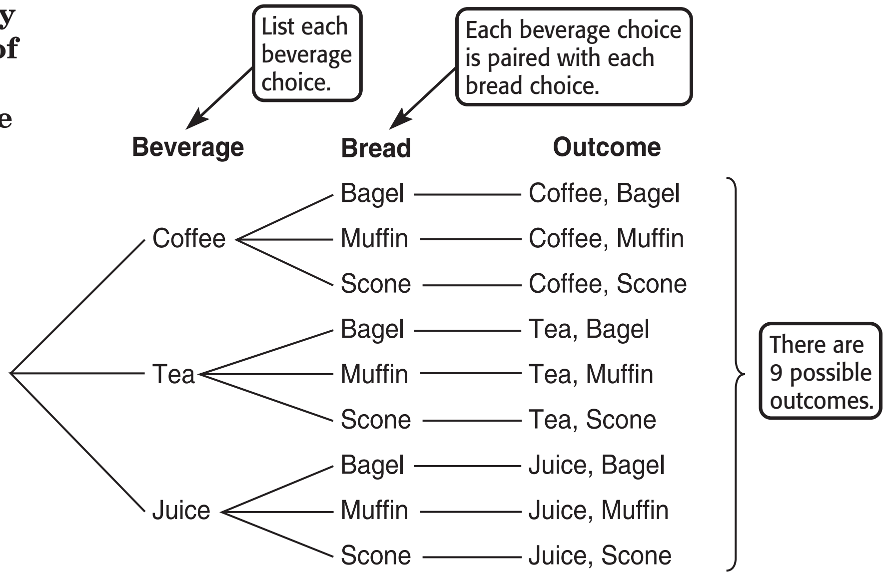
13-8 Study Guide and Intervention

Counting Outcomes

Counting Outcomes A **tree diagram** is a visual display used to find the number of outcomes given a number of choices. Another method that relates the number of outcomes to the number of choices is the **Fundamental Counting Principle**.

Fundamental Counting Principle	If event M can occur in m ways and is followed by event N that can occur in n ways, then the event M followed by N can occur in $m \cdot n$ ways

Example 1 **How many different combinations of beverage and bread can be made from 3 beverage choices and 3 bread choices? Draw a tree diagram to find the number of different combinations.**



Example 2 **Refer to Example 1. Use the Fundamental Counting Principle to find the total number of outcomes.**

beverage choices	times	bread choices	equals	total number of possible outcomes
3	×	3	=	9

Both the tree diagram and the Fundamental Counting Principle show that there are 9 possible combinations or outcomes when choosing from 3 beverage choices and 3 bread choices.

Exercises

For each situation, draw a tree diagram to find the number of outcomes.

1. A closet has a red top, a blue top, and a white top, and pants and a skirt.

2. Three pennies are flipped.

Use the Fundamental Counting Principle to find the total number of outcomes in each situation.

3. One six-sided number cube is rolled, and one card is drawn from a 52-card deck.

4. One letter and one digit 0–9 are randomly chosen.

13-8 Study Guide and Intervention *(continued)*

Counting Outcomes

Find the Probability of an Event When you know the number of outcomes, you can find the probability that an event will occur.

Example **ICE CREAM An ice cream parlor has a special where you can build your own sundae for $3. You are given a choice of chocolate, vanilla, or strawberry ice cream; sprinkles or nuts; and chocolate or caramel topping. What is the probability of randomly selecting vanilla ice cream with nuts and either chocolate or caramel topping?**

Use the Fundamental Counting Principle to find the number of outcomes.

ice cream choices	times	dry topping choices	times	wet topping choices	equals	total number of possible outcomes
3	×	2	×	2	=	12

Using a tree diagram, you can see that there are 2 possible outcomes for vanilla, nuts, and either chocolate or caramel topping.

So, the probability of randomly selecting vanilla ice cream with nuts and either chocolate or caramel topping is $\frac{2}{12}$ or $\frac{1}{6}$.

Exercises

1. **CLOTHES** A dresser has 4 shirts and 3 pants. If each shirt and pair of pants is a different color, what is the probability of randomly picking a blue shirt and black pants?

2. **CELL PHONES** There are 6 cell phones and 23 covers. If each cell phone is made by a different company, and each cover is different, what is the probability of randomly picking a Telecom phone with a green cover?

3. **COMPUTERS** A computer store offers 11 computers and 23 keyboards. If each computer and keyboard are made by different companies, what is the probability of randomly picking a Computz computer and a Language Inc. keyboard?

4. A nickel and a dime are flipped. What is the probability of getting tails, then heads?

5. A coin is tossed and a card is drawn from a 52-card deck. What is the probability of getting tails and the ten of diamonds?

6. Four coins are tossed. What is the probability of four tails?

13-9 Study Guide and Intervention

Permutations and Combinations

Use Permutations To find the number of permutations of a list of arranged items, find all the possible ways the order of the items can be arranged. Use the Fundamental Counting Prinicple to find the number of possible permutations.

Permutations	**Words**	An arrangement or listing in which order is important is called a **permutation**.
	Symbols	$P(m, n)$ means m number of choices taken n at a time.
	Example	$P(3, 2) = 3 \cdot 2 = 6$

Example 1 **SPORTS** **How many ways can the top five finishers be arranged in a 20-person cross-country race?**

Order is important.
So, this arrangement is a permutation.

20 runners
choose 5

20 choices for 1st place
19 choices for 2nd place
18 choices for 3rd place
17 choices for 4th place
16 choices for 5th place

$$P(20, 5) = 20 \cdot 19 \cdot 18 \cdot 17 \cdot 16$$
$$= 1{,}860{,}480 \text{ ways}$$

Example 2 **LOCKERS** **How many locker combinations can be made from the numbers 0 through 30 if each number is used only once?**

31 choices for the 1st number
30 choices remain for the 2nd number
29 choices remain for the 3rd number

$$P(31, 3) = 31 \cdot 30 \cdot 29$$
$$= 27{,}869$$

Exercises

Find the following permutations.

1. $P(4, 3)$ **2.** $P(11, 3)$ **3.** $P(6, 3)$

4. ACTORS How many ways can 15 actors fill 6 roles in a play?

5. MARATHON How many ways can 6 runners finish in first through sixth place in a marathon with 20 runners?

6. PASSWORDS How many different seven-digit passwords are possible using the digits 0–9 if each digit is used only once?

13-9 Study Guide and Intervention *(continued)*

Permutations and Combinations

Use Combinations A **combination** can be used to find the possible number of arrangements of items when order is *not* important. You can find the number of combinations of items by dividing the number of permutations of the set of items by the number of ways each smaller set can be arranged.

Example 1 **SANDWICHES How many different sandwiches can be made with 2 types of cheese if the choices are cheddar, Swiss, American, jack, and provolone?**

Order is not important. So, this arrangement is a combination.

Use the first letter of each cheese to list all of the permutations of the cheeses taken two at a time. Cross off arrangements that are the same as another one.

CS ~~SC~~ ~~AC~~ ~~JC~~ ~~PC~~ CJ SJ AJ ~~JA~~ ~~PA~~
CA SA ~~AS~~ ~~JS~~ ~~PS~~ CP SP AP JP ~~PJ~~

There are only 10 *different arrangements*. So, 10 sandwiches can be made using 2 types of cheese from a choice of five cheeses.

Example 2 **SCHOOL In a science class with 42 students, how many 3-person lab teams can be formed?**

Order is not important.
So, this arrangement is a combination.

From 42 students, take 3 at a time.

$$C(42, 3) = \frac{P(42, 3)}{3 \cdot 2 \cdot 1}$$

There are $3 \cdot 2 \cdot 1$ ways to create a 3-person team.

$$= \frac{42 \cdot 41 \cdot 40}{6} \text{ or } 11{,}480 \text{ lab teams}$$

Exercises

Find the following combinations.

1. $C(4, 3)$ **2.** $C(11, 3)$ **3.** $C(6, 3)$

4. BOOKS How many ways can 5 books be borrowed from a collection of 40 books?

5. JOBS A telemarketing firm has 35 applicants for 8 identical entry-level positions. How many ways can the firm choose 8 employees?

6. FOOD A pizza place sends neighbors a coupon for a 4-topping pizza of any size. If the pizzeria has 15 toppings and 3 sizes to choose from, how many possible pizzas could be purchased using the coupon?

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13-10 Study Guide and Intervention

Probability of Compound Events

Probability of Two Independent Events	**Words**	The probability of two independent events is found by multiplying the probability of the first event by the probability of the second event.
	Symbols	$P(A \text{ and } B) = P(A) \cdot P(B)$
Probability of Two Dependent Events	**Words**	If two events, A and B, are dependent, then the probability of events occurring is the product of the probability of A and the probability of B after A occurs.
	Symbols	$P(A \text{ and } B) = P(A) \cdot P(B \text{ following } A)$

Example 1 **GAMES A card is drawn from a standard deck of 52 cards. The card is replaced and another is drawn. Find the probability if the first card is the 3 of hearts and the second card is the 2 of clubs.**

Since the first card is replaced, the events are independent.

$$P(\text{3 of hearts and 2 of clubs}) = P(\text{3 of hearts}) \cdot P(\text{2 of clubs})$$
$$= \frac{1}{52} \cdot \frac{1}{52}$$
$$= \frac{1}{2704}$$

The probability is $\frac{1}{2704}$.

Example 2 **PRIZES A prize bag contains 4 whistles, 3 yo-yos, and 9 pencils. Each winner of a game randomly selects and keeps one of the prizes. What is the probability that a whistle is chosen from the bag, followed by a yo-yo?**

Since the first prize is kept by the winner, the first event affects the second event. These are dependent events.

$P(\text{the first prize is a whistle}) = \frac{4}{16}$ ← $\frac{\text{number of whistles}}{\text{total number of prizes}}$

$P(\text{the second prize is a yo-yo}) = \frac{3}{15}$ ← $\frac{\text{number of yo-yos}}{\text{total number of prizes after one prize is removed}}$

$P(\text{whistle, then yo-yo}) = \frac{4}{16} \cdot \frac{3}{15} = \frac{12}{240} \text{ or } \frac{1}{20}$

Exercises

A card is drawn from a standard deck of cards. The card is replaced and a second card is drawn. Find each probability.

1. P(4 and 8)

2. P(queen of hearts and 10)

3. P(4 of spades and 7 of clubs)

4. P(red jack and black ace)

A card is drawn from a standard deck of cards. The card is *not* replaced and a second card is drawn. Find each probability.

5. P(4 and 8)

6. P(queen of hearts and 10)

7. P(4 of spades and 7 of clubs)

8. P(red jack and black ace)

NAME ______________________________ DATE ____________ PERIOD ____________

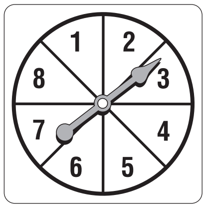
13-10 Study Guide and Intervention *(continued)*

Probability of Compound Events

Mutually Exclusive Events If two events cannot happen at the same time, they are said to be **mutually exclusive events**. If you roll two six-sided number cubes, you cannot roll both a sum of 7 and doubles at the same time. The probability of mutually exclusive events can be found by adding.

Probability of Mutually Exclusive Events		
	Words	The probability of one or the other of two **mutually exclusive events** can be found by adding the probability of the first event to the probability of the second event.
	Symbols	$P(A \text{ or } B) = P(A) + P(B)$

Example 1 **The spinner at the right is spun. What is the probability that the spinner will stop on 7 or an even number?**



The events are mutually exclusive because the spinner cannot stop on both 7 and an even number at the same time.

$P(7 \text{ or even}) = P(7) + P(\text{even}) = \frac{1}{8} + \frac{1}{6} + = \frac{5}{8}$

The probability that the spinner will stop on 7 or an even number is $\frac{5}{8}$.

Example 2 **A six-sided number cube is rolled. What is the probability of rolling a multiple of three or the number 5?**

There are only 2 multiples of three on a six-sided number cube, 3 or 6. The cube cannot land on 3 or 6 and 5 at the same time.

$P(\text{multiple of three or } 5) = P(\text{multiple of three}) + P(5) = \frac{1}{3} + \frac{1}{6} = \frac{1}{2}$

The probability that a six-sided number cube will land on a multiple of three or 5 is $\frac{1}{2}$.

Exercises

Refer to the spinner in Example 1. Find each probability.

1. $P(2 \text{ or odd})$

2. $P(\text{prime or } 1)$

Two six-sided number cubes are rolled at the same time. Find each probability.

3. $P(\text{the sum is 7 or the sum is 4})$

4. $P(\text{the sum is odd or the sum is even})$

A card is drawn from a standard deck of cards. Find each probability.

5. $P(\text{queen of clubs or a red card})$

6. $P(\text{queen of hearts or } 10)$